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TECHNICAL NOTE

An adaptable problem-space-based search method for flexible flow line scheduling

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Leon and Ramamoorthy (1997) developed an adaptable problem-space-based search method. Part of that work included development of a lower bound and comparison of makespans using real-world data. In this note we improve their lower bound and show that the Leon and Ramamoorthy search method found better makespan solutions than their lower bound indicated, including some optimal makespans.

1. Introduction

A flexible flow line is a manufacturing system consisting of several serial stages with at least one identical machine in each stage. Jobs move through the stages in order, but may be able to skip stages depending on the process plan. The following notation will be used in this note:

R = number of stages;
M_s = number of machines in stage s;
d_{is} = processing time of job i in stage s.

Section 2 of this note improves the lower bound from Leon and Ramamoorthy (1997) for the optimal makespan in a flexible flow line. Section 3 demonstrates the improved lower bounds using data from the paper of Wittrock (1988) and the makespans found in Leon and Ramamoorthy (1997) to show that Leon and Ramamoorthy's methods worked better than reported. Section 4 concludes the note.

2. Improved lower bounds

Leon and Ramamoorthy (1997) present the following lower bound. To remove ambiguity, we correct a minor typo by replacing subscript "s" with "r" in the middle term.

$$LB = \max_{r=1,\dots,R}\left\{\sum_{s=1}^{r-1}\min_i\{d_{is}\} + \sum_i d_{ir}/M_r + \sum_{s=r+1}^{R}\min_i\{d_{is}\}\right\}. \quad (1)$$

LB acknowledges that each stage provides a lower bound equal to its earliest activation plus minimum active duration plus time to clear subsequent stages. LB can be strengthened by observing that work at a given stage cannot begin until the first job reaches that stage. The time for a job to reach a given stage is bounded by the sum of its processing times for all stages up to that stage. For any set of d_{is},

$$\min_i \sum_{s=1}^{r} d_{is} \geq \sum_{s=1}^{r}\min_i\{d_{is}\}.$$

The same follows for the time until the last job finishes processing after a given stage. LB is strengthened as follows:

$$LB^1 = \max_{r=1,\dots,R}\left\{\min_i \sum_{s=1}^{r-1} d_{is} + \sum_i d_{ir}/M_r + \min_i \sum_{s=r+1}^{R} d_{is}\right\}. \quad (2)$$

It is understood that the

$$\min_i,$$

refers only to the jobs i with $d_{ir} > 0$. LB^1 can be strengthened by observing that work at a given stage can only begin at one machine when the first job arrives. The second machine is idle until the second job arrives, and so on. By allocating the idle time across all machines at the given stage, we strengthen LB^1. The notation

$$\text{“}\min_{[k]}\text{”},$$

will be used to indicate the $(k+1)$st from the lowest value and

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Table 1. Comparison of local search average makespans from Leon and Ramamoorthy (1997) with lower bounds

				Percent deviation								
				LB			LB^1			LB^2		
Day	LB	LB^1	LB^2	SNS	SDS	FIS	SNS	SDS	FIS	SNS	SDS	FIS
1	720	734	734	5.64	6.53	6.53	3.62	4.50	4.50	3.62	4.50	4.50
2	728	767	767	5.58	6.02	6.18	0.21	0.63	0.78	0.21	0.63	0.78
3	741	761	761	4.24	5.21	4.76	1.50	2.44	2.01	1.50	2.44	2.01
4	713	761	761	8.39	8.78	8.56	1.55	1.92	1.71	1.55	1.92	1.71
5	941	961	961	2.13	2.13	2.30	0.00	0.00	0.17	0.00	0.00	0.17
6	631	648	649	5.59	5.63	6.15	2.82	2.85	3.36	2.67	2.70	3.20

$$\min_{[0]} \equiv \min.$$

For example, given a list of values {2, 5, 7, 8, 9},

$$\min_{[1]} = 5.$$

LB^1 is strengthened as follows:

$$LB^2 = \max_{r=1,\dots,R} \left\{ \min_i \sum_{s=1}^{r-1} d_{is} + \sum_i d_{ir}/M_r + \min_i \sum_{s=r+1}^{R} d_{is} + \frac{1}{M_r} \sum_{k=1}^{M_r-1} \left(\min_{i}{}_{[k]} \sum_{s=1}^{r-1} d_{is} - \min_i \sum_{s=1}^{r-1} d_{is} \right) \right\}. \quad (3)$$

3. Computational results

Leon and Ramamoorthy (1997) use data from the paper of Wittrock (1988) for computational comparisons. In this section, we also use this data to demonstrate the improved lower bounds. Table 1 shows the percent deviation between each lower bound and the local search solutions from Leon and Ramamoorthy (1997). The percent deviation is 100 × (value − lower bound)/lower bound, where value is the average of 10 runs with randomly generated neighborhoods. LB^1 is greater than LB in each of these cases. LB^2 is greater than LB^1 in only one case. This is because of the large number of jobs with the same processing times in the data set. We can see that while Leon and Ramamoorthy claimed to be very close to the optimal makespan, they were in fact closer than they thought. We verify that they found the optimal makespan for day 5 using both the SNS and SDS methods for generating neighborhoods. The original average gap of 5.26% for the SNS procedure has been reduced to 1.59 with LB^2.

4. Conclusions

In this note we have improved a lower bound on the makespan for a flexible flow line and shown that Leon and Ramamoorthy's search methods performed better than previously reported.

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Biographies

Mary E. Kurz is a Ph.D. candidate in Systems and Industrial Engineering at The University of Arizona. Ms. Kurz received a B.S. and M.S. in Systems Engineering from The University of Arizona. She is a Student Member of the IIE and INFORMS and a member of Tau Beta Pi. She holds a Department of Energy Predoctoral Fellowship in Integrated Manufacturing and Processing.

Ronald G. Askin is a Professor and Department Head of Systems and Industrial Engineering at The University of Arizona. Dr. Askin received a B.S. in Industrial Engineering from Lehigh University, and an M.S. in Operations Research and Ph.D. in Industrial & Systems Engineering from Georgia Institute of Technology. He is a Fellow of the IIE, and an active member of INFORMS, ASQC, and SME. Currently he serves on the editorial boards of the *IIE Transactions on Design and Manufacturing* and *IIE Solutions* and serves as President of the INFORMS Manufacturing and Service Operations Management Society (MSOM). He has previously served as chair of the ORSA Technical Section on Manufacturing Management and the Statistics Division of ASQC. He has authored or co-authored over 80 professional publications, primarily on the application of operations research and statistical methods to the design and analysis of production systems. His current research concentrates on developing integrated models for facilities planning, production planning, operation scheduling, worker scheduling, and material flow control. Dr. Askin co-authored the text *Modeling and Analysis of Manufacturing Systems* which was awarded the 1994 IIE Joint Publishers Book of the Year Award. Other awards he has received include the *IIE Transactions on Design and Manufacturing* Best Paper Award (co-author), the Shingo Award for Excellence in Manufacturing Research, *IIE Transactions* Development and Applications Award (co-author), the ASEE/IIE Eugene L. Grant Award (co-author), and a National Science Foundation Presidential Young Investigator Award. Dr. Askin has consulted with a variety of industrial companies in the areas of scheduling, facilities planning, quality improvement, and performance evaluation of manufacturing systems.

Contributed by the Scheduling Department