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Multi-level inventory management decisions with transportation cost consideration

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ABSTRACT

In this article we address specific inventory management decisions with transportation cost consideration in a multi-level environment consisting of a supplier–warehouse–retailers. We develop two models – namely, decentralized ordering model and centralized ordering model to investigate the effect of collective ordering by retailers on the total inventory cost of the system. A numerical study shows that the proposed model is robust and generates reasonable cost savings. The models have potential in several multi-level applications such as fresh or frozen food delivery to stores of different supermarkets or the supply of medicine to a number of hospitals from a wholesaler.

1. Introduction

Inventory models developed for the deployment of stock in a source-deliver supply chain take complexities of multi-level distribution systems into account but usually fall short in considering transportation cost. Increasing oil costs, highway congestion, increasing cost of short-hauls, and consolidation of distribution centers are examples of problems that may increase transportation costs. Source-deliver chains desire not only agility but also lower costs in the network. According to Swenseth and Godfrey (2002), upwards of 50% of total logistics costs can be attributed to transportation.

Classical inventory management strategies are not applicable and many companies are searching for new supply chain inventory strategies to contain the rising costs of transportation, find offsetting savings, maintain low inventories, and also ensure on-time deliveries. Even though there is considerable research on inventory control and transportation management, much less is available on the combined problem. The traditional Economic Order Quantity (EOQ) model captures only the trade-off between inventory carrying cost and setup or ordering cost. In this respect the transportation cost usually is neglected or included in another cost such as setup cost.

In this paper, a single item inventory model is considered with several retailers, one warehouse and one supplier. We develop two new models – namely (1) decentralized ordering and (2) centralized ordering. The decentralized ordering model is a constructive extension of the classic optimal Economic Order Quantity (EOQ) model. In the presented model, we consider two components in the per order cost: a fixed ordering cost which excludes transportation costs, and a discrete transportation cost. We try to minimize the total inventory cost while considering the transportation cost as a part of it. This cost is imposed whether a truck is fully loaded or only partly loaded. Hence, we integrate inventory and transportation management into one mathematical model. We develop the transportation cost based on a practical application. Since the number of trucks required is always a positive integer, the transportation cost is a discrete function of the order quantity. We also

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search for the optimal strategy of the warehouse (i.e., how often to place orders) to determine the optimal review period. Simple algorithms are also presented to compute the optimal order quantity for the retailers and the optimal review for the warehouse.

Based on our numerical experiment, we show that the transportation cost contains a considerable percentage of the total inventory cost. There are new elements in our model which distinguish it from other extensions to the traditional EOQ models with regard to transportation cost; these features are discussed in the next section.

In the centralized ordering model, we propose a collective form of ordering by retailers and minimize the inventory cost of the retailers and the warehouse jointly. Here retailers observe their customers' demand, and then collaborate to explore the optimal joint ordering, and send it to the warehouse. A continuous review model and a simple algorithm are applied to determine the optimal order quantity and optimal re-order point of the system.

We show that total cost of the model can be decreased through collaboration among the retailers and the warehouse. Numerical examples are used to solve both models and compare the cost savings.

The paper is structured into five sections. Section 2 presents a literature review dealing with inventory models and transportation cost elements. Section 3 briefs the boundary of supply chain inventory problem, i.e. the inventory policy adopted for retailers and the warehouse. Section 4 formulates the decentralized ordering policy. Then in Section 5, the model is extended to study the effect of a centralized (collective) form of ordering by the downstream entities and combined delivery on total costs. The benefits of transportation considerations are then highlighted through numerical studies in Section 6. A sensitivity analysis is conducted in this section as well to evaluate the outcomes of numerical result for different values. Finally, Section 7 presents brief conclusions.

2. Literature overview

Several attempts have been made to extend the EOQ model to different conditions. For this purpose, a few authors incorporated transportation costs into the lot-size determination analysis. Baumol and Vinod (1970) considered an inventory-theoretic model of freight transport to determine order quantity and transportation. Their objective was to minimize the total transportation, ordering, and carrying costs. The model however considered a per unit constant transportation cost. Das (1974) worked on the same model with a few different assumptions. His model considered the determination of a fixed order quantity and safety stock sizes from the order size. Buffa and Reynolds (1977) extended these works by adding stock-out costs and shipment discounts based on minimum full truckloads. Burwell et al. (1997) developed a model for determining the reseller's lot-size and price assuming that there are freight and all-unit quantity discount breakpoints in the pricing schedule offered by the supplier. Blumenfeld et al. (1987) developed a decision support tool for the analysis of the logistics operations at General Motors that resulted in a 26% reduction in logistics costs. While allowing the analysis and models to be as simple as possible, the authors developed a tool that allowed the Delco Electronics Division to examine the impact on total corporate cost due to different shipping strategies for its products. The authors stated that the minimization of total network cost required the simultaneous determination of optimal routes and shipment sizes and they focused on analyzing the trade-off between inventory and transportation costs. Results obtained from the research are reported in a series of papers (Blumenfeld et al., 1985; Burns et al., 1985).

Gupta (1992) considered a situation in which a fixed cost is incurred for a transport mode such as a truck that has a fixed load capacity. He developed a model to determine the optimal lot-size, which minimizes the sum of the inventory holding, ordering and transportation costs. Zhao et al. (2004) addressed the problem of evaluating the optimal ordering quantity in a supplier–customer model by considering the transportation cost. They made a trade-off among production, inventory and transportation costs where transportation cost involved fixed and variable costs.

The approach that we propose in Section 4 (the decentralized ordering model) is similar to Gupta's (1992) but with some extensions and differences. First, we use a multi-level (one warehouse–multiple retailers) model with a determined inventory policy for each element. These policies facilitate safety stock at the retailers and warehouse. Second, we make a more precise inventory cost estimation for carrying cost by adding the lead-time and the demand during the lead-time and by including the distance between warehouse and retailers as a factor in the transportation cost. Third, the demands are assumed to be stochastic. Finally, we consider both fixed and variable cost of transportation in our model while a fixed transportation cost was only addressed in Gupta's research.

In the area of centralized (collective) ordering at downstream entities, to the best of our knowledge, little research has been carried out where many firms (retailers or manufacturers) collaborate and send their order for one item as a combined-order to one supplier. However, there are many research publications on the Joint Replenishment Problem (JRP) in which several items are replenished at a single stocking point. A complete definition of JRP is available in Goyal (1973, 1974) and Goyal and Satir (1989).

3. Structure of source-deliver inventory decisions

A two-level supply chain consisting of a warehouse (distribution center) and N retailers is considered in our model (Fig. 1).

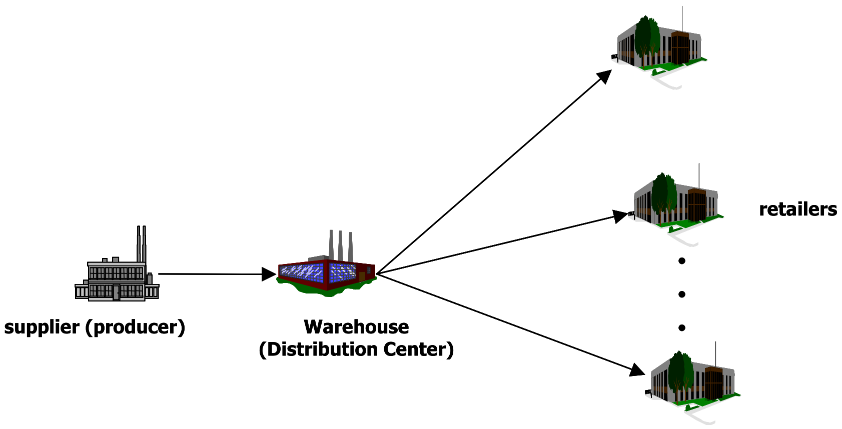

Fig. 1. The structure of the model.

Each retailer and the warehouse have a set of control parameters that affects the performance of other components. In the decentralized ordering model each individual level tries to optimize its own total cost. The objective is

$$Z^d = TC(Q_1^*) + TC(Q_2^*) + \cdots + TC(Q_N^*) + TC_w^*$$

where $TC(Q_j^*)$ is the total optimal cost for retailer j and TC_w is the total cost for the warehouse. In the centralized model, the total cost of the retailers and warehouse is optimized simultaneously, so we will have:

$$\text{Minimize } Z^c \left(= \sum_{j=1}^{N} TC_j(Q_j) + TC_w \right)$$

In our models the following notations are used:

N	number of retailers
j	retailer index ($j = 1, 2 \ldots N$)
Q_j	replenishment order quantity in units of retailer j
A_j, A_w	fixed cost of order or cost per order event at the retailers and warehouse (not per unit) ($)
V_j, V_w	variable purchase cost of item at retailer j and warehouse ($)
D_j, D_w	demand quantity (yearly) observed by retailer j and the warehouse
d_{wj}, d_{sw}	traveling distance from warehouse to retailer j and from supplier to warehouse (km)
r_j, r_w	carrying charge in % of unit value at retailer j and at the warehouse (per year)
S_j, S_w	re-order point at retailer j and at the warehouse
P_j, P_w	service level at retailer j and at the warehouse
K_j, K_w	safety factor at retailer j and at the warehouse
L_{wj}, L_{sw}	lead-time from warehouse to retailer j and supplier to warehouse
VC	truck capacity (items)
ρ	density of retailers (retailers per square km)
m	maximum number of stops made by a truck
B_w	fixed cost per stock-out occasion ($) at warehouse
g	number of trucks
R_w	review period in the warehouse interval between orders
α_w, α_s	fixed cost of transportation per shipment (or per order) from warehouse to retailer j and supplier to warehouse (same value for all retailers)
t_w, t_s	variable cost of transportation from warehouse to retailer j and supplier to warehouse
SS_j	safety stock at retailer j

Throughout this study, we assume that the demand is stationary and that the forecast errors are normally distributed. We make these assumptions for three reasons. First, empirically in many cases the normal distribution provides a better fit than other distributions. Second, particularly if the lead-time is long relative to the base forecasting period, forecast errors in many periods are added together, so we would expect a normal distribution through the Central Limit Theorem. Lastly, the normal distribution leads to analytically good results. We assume that retailers offer one product type and demand arrives at each retailer for this product. All retailers use the same forecasting technique. The demand which retailer j will face is

$$D_j = \mu_j + \varepsilon_j \tag{1}$$

where D_j is the forecast of customer demand by retailer j, $\mu_j > 0$ is the average demand per period (yearly with limited variation) and the ε_j are *i.i.d.* normally distributed error terms with mean zero and variance σ_j^2. We further assume that σ_j^2 is significantly smaller than μ_j so that the probability of negative demand is negligible. In this case $E(D_j) = \mu_j$ and $Var(D_j) = \sigma_j^2$. The above statement results in the following lemma:

Proposition 1. $D_w \sim N\left(\mu_w = \sum_{j=1}^{N}\mu_j, \sigma_w^2 = \sum_{j=1}^{N}\sigma_j^2\right)$.

The proof is straightforward due to the fact that the retailers' forecast errors are normally distributed and using the theory of Linear Combination of Random Variables.

3.1. Inventory policy of the retailers

We assume a continuous review, (S, Q), policy for all of the retailers. Initial inventory position is assumed to be zero. When the inventory position (IP_j) at each retailer decreases to or below S_j, an amount Q_j will be ordered. For convenience, we assume that all of the retailers use a similar service level, say P_1 (see Silver et al., 1998) that determines the probability of no stock-out per order cycle. In addition, lost sales are not considered in the model. The resulting inventory position will be strictly larger than S_j and smaller than or equal to $S_j + Q_j$. Finally we assume that lead-time demand is normally distributed with average $\mu_{jL_{wj}}$ and variance $\sigma^2_{jL_{wj}}$ at each retailer j. We define $\mu_{jL_{wj}}$ as the expected demand over a replenishment lead-time, as follows:

$$E(D_{jL_{wj}}) = \mu_{jL_{wj}} = L_{wj}\mu_j \tag{2}$$

Since the variation of the lead-time from the warehouse to each retailer is zero, $\sigma_{jL_{wj}}$ is calculated as follows:

$$\sigma_{jL_{wj}} = \sqrt{L_{wj}\sigma_j^2} \tag{3}$$

Ultimately, we can compute the re-order point of each retailer as (see Silver et al., 1998):

$$P(D_{jL_{wj}} < S_j) = P_j = \Phi\left(\frac{S_j - \mu_{jL_{wj}}}{\sigma_{jL_{wj}}}\right) = \Phi\left(\frac{SS_j}{\sigma_{jL_{wj}}}\right) = \Phi(K_j) \tag{4}$$

Since $S_j - \mu_{jL_{WJ}} = SS_j$, we develop an expression for S_j as follows:

$$S_j = K_j\sigma_{jL_{wj}} + \mu_{jL_{Wj}} \tag{5}$$

In the following section we discuss the inventory policy of the warehouse.

3.2. Inventory policy of the warehouse

Suppose that the warehouse applies a periodic-review (*R, S*) inventory control policy in which at every *R* units of time, the order-up-to point, S_w, is estimated from the observed demand. Additionally, the lead-time from the supplier to the warehouse is assumed to be fixed. As mentioned in the previous section, the retailer demand is normally distributed. The lead-time between the warehouse and each retailer is fixed, given the distance between the two.

Since the warehouse reviews its inventory level periodically, it may order at every fixed period of time in order to maintain an inventory position at a predefined base stock level S_w. This is a fair assumption considering the fact that the warehouse as a wholesaler/distributor has good knowledge of the retailers, their ordering process and the demand of the customers that the retailers observe. Following the analytical technique in Section 3.1, we assume that the warehouse uses a simple service criteria P_w (based on P_1). We define the re-order point at the warehouse using the following:

$$1 - P_w = 1 - \Phi\left(\frac{S_w - \mu_{w(R_w+L_{sw})}}{\sigma_{w(R_w+L_{sw})}}\right) \tag{6}$$

or,

$$P_w = \Phi\left(\frac{S_w - \mu_{w(R_w+L_{sw})}}{\sigma_{w(R_w+L_{sw})}}\right) = \Phi(K_w) \tag{7}$$

We develop the expression for the re-order point at the warehouse as

$$S_w = \mu_{w(R_w+L_{sw})} + K_w\sigma_{w(R_w+L_{sw})} \tag{8}$$

where $\mu_{w(R_w+L_{sw})}$ is the expected demand over a review period in the warehouse and $\sigma_{w(R_w+L_{sw})}$ is the variance of the supply time to the warehouse. We estimate these terms as described in the following.

We assume that the warehouse uses a periodic-review of inventory. In this respect whenever the inventory position (IP_w) at a review moment is lower than S_w, an order will be placed by the warehouse to the supplier to maintain an inventory position of at least S_w. The expected value of IP_w just before the order being placed is

$$E(IP_w) = S_w - (E(D_{R_w+L_{sw}}) \tag{9}$$

where $E(D_{R_w+L_{sw}})$ indicates the expected total retailer demand during the review period R_w. When $IP_w < S_w$, the warehouse sends an order to the supplier. The expected warehouse order size is

$$E(Q_w) = S_w - E(IP_w) \tag{10}$$

Given (9) and (10) we find:

$$E(Q_w) = E(D_{R_w+L_{sw}}) = \mu_{w(R_w+L_{sw})} \tag{11}$$

Due to the independence of the retailer orders and also because it is not certain that the retailers will order at the same time, we have to estimate $\mu_{w(R_w+L_{sw})}$. The frequency of each retailer demand is μ_j/Q_j. Computing the amount of the order during the review period of the warehouse, we find

$$\mu_{w(R_w+L_{sw})} = (R_w + L_{sw})\sum_{j=1}^{N}\left(Q_j\frac{\mu_j}{Q_j}\right)$$

or

$$\mu_{w(R_w+L_{sw})} = (R_w + L_{sw})\sum_{j=1}^{N}\mu_j \tag{12}$$

The variance of the supply time to the warehouse is

$$\sigma_{w(R_w+L_{sw})} = \sqrt{(R_w + L_{sw})\sum_{j=1}^{N}\sigma_j^2} \tag{13}$$

The above definitions for the expected demand and variance are meant only for decentralized ordering policy. Later, we redefine these terms for the centralized model.

In order to avoid the use of the standard normal, a simple expression to determine the safety factor for the periodic-review case (K_w) has been proposed,. This approximation is as follows (Johnson et al., 1996):

$$K_w \approx \left(\frac{1}{2}\sqrt{\pi/2}\right)ln\left(\frac{P_w}{1-P_w}\right) \tag{14}$$

Johnson et al. evaluated the above expression via an extensive set of simulations and found that it is fairly robust, providing feasible results.

4. Decentralized ordering optimization

In this section we consider the warehouse and retailers to be distinct entities making individual decisions. We compute the total cost of the warehouse and each retailer in following.

4.1. Retailer model

Here we consider the case that each retailer determines its own Economic Order Quantity and optimal cost. We assume that the retailer's costs include transportation costs, the cost of replenishment, and carrying cost. For convenience, we neglect the cost of stock-out. Considering the warehouse inventory policy, the chance of retailers' stock-out of the retailers is very limited. The total cost to each retailer is defined as follows:

$$TC_j(Q_j) = C_{R_j} + C_{C_j} + C_{T_j}$$

The first term indicates the replenishment cost and can be determined as:

$$C_{R_j} = A_j\frac{D_j}{Q_j} \tag{15}$$

The second term indicates the carrying cost and can be determined as:

$$C_{C_j} = \left(\frac{Q_j}{2} + K_j\sigma_{jL_{wj}}\right)V_j r_j \tag{16}$$

Finally the third term indicates the transportation cost. To compute the transportation cost, we first assume that the contract between the retailers and the warehouse is not Free On Board (FOB) so the warehouse is not responsible for the transportation cost (the same policy is assumed between the warehouse and supplier). Recall that the retailer demand is defined as a yearly value. The transportation cost is decomposed into fixed and variable costs. Given D_j and Q_j for each retailer j, the number of the trips from the warehouse to retailer j is estimated as $\frac{D_j}{Q_j}$. Furthermore, $g_j = \left\lceil \frac{Q_j}{VC} \right\rceil$ indicates the number of trucks needed per replenishment. Subsequently, the transportation cost can be estimated as

$$C_{T_j} = \left(\alpha_w + t_w \left\lceil \frac{Q_j}{VC} \right\rceil d_{wj} \right) \frac{D_j}{Q_j} \tag{17}$$

Given (15)–(17), the total cost of each retailer is estimated as

$$TC_j(Q_j) = A_j \frac{D_j}{Q_j} + \left(\frac{Q_j}{2} + K_j \sigma_{jL_{wj}} \right) V_j r_j + \left(\alpha_w + t_w \left\lceil \frac{Q_j}{VC} \right\rceil d_{wj} \right) \frac{D_j}{Q_j}$$

With the substitution $g_j = \left\lceil \frac{Q_j}{VC} \right\rceil$, we rewrite the total cost for retailer j as

$$TC_j(Q_j) = A_j \frac{D_j}{Q_j} + \left(\frac{Q_j}{2} + K_j \sigma_{jL_{wj}} \right) V_j r_j + (\alpha_w + t_w g_j d_{wj}) \frac{D_j}{Q_j} \tag{18}$$

The function $TC_j(Q_j)$ is a convex function of Q_j that can be re-written as

$$TC_j(Q_j) = \begin{cases} A_j \frac{D_j}{Q_j} + \left(\frac{Q_j}{2} + K_j \sigma_{jLwj} \right) V_r r_j + (\alpha_w + t_w g_j d_{wj}) \frac{D_j}{Q_j} & (g_j - 1)VC < Q_j < g_j VC \\ A_j \frac{D_j}{g_j VC} + \left(\frac{g_j VC}{2} + K_j \sigma_{jLwj} \right) V_r r_j + (\alpha_w + t_w g_j d_{wj}) \frac{D_j}{g_j VC} & Q_j = g_j VC \end{cases} \tag{19}$$

Eq. (19) indicates the fact that we have a piecewise function since $TC_j(Q_j)$ is given by different expressions on various intervals. To minimize the cost of the model, we should find the optimal value for Q_j by solving the equation $\frac{dTC_j(Q_j)}{dQ_j} = 0$. If $TC_j(Q_j)$ is continuous at Q_j, then the derivative of $TC_j(Q_j)$ exists. In our case the following lemma is valid.

Proposition 2. *TC(j) is continuous at j = Q.*

Proof 1. This is true if and only if

$$\lim_{j \to Q} TC(j) = TC(Q)$$

Moreover, by definition a function is piecewise differentiable if it is differentiable through a specified domain, except for a discrete set of points (such as $Q_j = g_j VC$). Therefore, there are two cases for Q_j: Q_j is on the line or on the step points.Case 1 ($(g_j - 1)VC < Q_j < g_j VC$):

$$\frac{dTC_j(Q_j)}{dQ_j} = 0 \Rightarrow -A_j \frac{D_j}{Q_j^2} + \frac{V_r r_j}{2} - \alpha_w \frac{D_j}{Q_j^2} - \frac{t_w d_{wj} g_j D_j}{Q_j^2} = 0$$

Subsequently,

$$Q_j^g = \sqrt{\frac{2D_j(A_j + \alpha_w + t_w g_j d_{wj})}{V_r r_j}} \tag{20}$$

and,

$$TC_j(Q_j^g) = \sqrt{2D_j V_r r_j (A_j + \alpha_w + t_w g_j d_{wj})} + K_j \sigma_{jLwj} V_r r_j \tag{21}$$

Case 2 ($Q_j = g_j VC$):

$$TC_j(Q_j^g) = A_j \frac{D_j}{g_j VC} + \left(\frac{g_j VC}{2} + K_j \sigma_{jLwj} \right) V_r r_j + (\alpha_w + t_w g_j d_{wj}) \frac{D_j}{g_j VC}$$

or

$$TC_j(Q_j^g) = A_j \frac{D_j}{g_j VC} + \left(\frac{g_j VC}{2} + K_j \sigma_{jLwj} \right) V_r r_j + \alpha_w \frac{D_j}{g_j VC} + t_w d_{wj} \frac{D_j}{VC} \tag{22}$$

□

We develop an algorithm to find Q_j^* and $TC_j(Q_j^*)$. The idea behind this algorithm is similar to the classical EOQ model with the quantity discount. By applying this algorithm first we explore the feasible Q_j's on the lines and compute the corresponding costs. In addition, we calculate the cost at the step points. By comparing the obtained costs, we can determine the minimum cost and the corresponding optimal order quantity for each retailer. In this respect, suppose that in (18) $g_j = 1$, and we find:

$$TC_j(Q_j^1) = A_j \frac{D_j}{Q_j} + \left(\frac{Q_j}{2} + K_j \sigma_{jLwj} \right) V_r r_j + (\alpha_w + t_w d_{wj}) \frac{D_j}{Q_j} \tag{23}$$

Given (20), the optimal order quantity can be computed by the following relation:

$$Q_j^1 = \sqrt{\frac{2D_j(A_j + \alpha_w + t_w d_{wj})}{(V_r r_j)}} \tag{24}$$

If $Q_j^1 < VC$, then $Q_j^* = Q_j^1$ and the corresponding total cost can be computed by using (21) or else if $Q_j^1 = VC$, then $Q_j^* = VC$ and the corresponding total cost can be computed by using (23). We set the result of (23) as the upper bound (UB) of the total annual cost. Ordering with $g_j > 1$ trucks will not be feasible when the following relation holds:

$$\sqrt{2D_j V_r r_j (A_j + \alpha_w + t_w g_j d_{wj})} + K_j \sigma_{jLwj} V_r r_j > UB_j.$$

Therefore,

$$g_j > \frac{\left[\frac{(UB_j - K_j \sigma_{jLwj} V_r r_j)^2}{2D_j V_r r_j} - A_j - \alpha_w\right]}{t_w d_{wj}} = e_j \tag{25}$$

Based on the above analysis, the optimal order quantities associated with the total cost function can be computed by applying following algorithm:

Algorithm 1.

Step 1. For each retailer j
Step 2. Let $g_j = 1$, Compute the Q_j^1 by using Eq. (24).
Step 3. If $Q_j^1 < VC$ then set $Q_j^* = Q_j^1$ as the optimal order quantity, compute Eq. (21) for $g_j = 1$ and go to Step 7.
Step 4. Else set $Q_j^* = VC$ then compute Eq. (23) and set the result as UB_j.
Step 5. Compute e_j by using Eq. (25).
Step 6. For $g_j = 2$ to $\lfloor e_j \rfloor$
Step 6.1. Compute Eq. (20). If $(g_j - 1)VC < Q_j^g < g_j VC$ then set $Q_j^* = Q_j^g$ and compute Eq. (21), else go to step 6.3.
Step 6.2. Set UB_j' as the minimum of all computed $TC_j(Q_j^g)$ in the previous step and set the corresponding Q_j^g as Q_j^*.
Step 6.3. Set $Q_j^g = gVC$ and compute Eq. (22).
Step 6.4. Set UB_j'' as the minimum of all computed $TC_j(Q_j^g)$ in step 6.3 and set the corresponding $Q_j^g(Q_j^g = gVC)$ as Q_j^*.
Step 6.5. Find the minimum of UB_j, UB_j' and UB_j''. Write the result as optimal total cost $(TC_j(Q_j^*))$ and the corresponding Q_j^*.
Step 7. End

4.2. Warehouse model

We search for the optimal strategy of the warehouse (i.e., how often to place orders) that will minimize the costs of total inventory cost. Here the total cost is comprised of replenishment cost, carrying cost, shortage cost, and transportation cost. Hence, the total cost of the warehouse can be defined as follows:

$$TC_w = C_{R_w} + C_{C_w} + C_{s_w} + C_{T_w}$$

where C_{R_w}, C_{C_w} and C_{s_w} can be determined as follows

$$C_{R_w} = \frac{A_w}{R_w} \tag{26}$$

$$C_{C_w} = \left(\frac{\mu_{w(R_w+L_{sw})}}{2} + K_w.\sigma_{w(R_w+L_{sw})}\right) V_w r_w \tag{27}$$

$$C_{s_w} = \frac{B_w}{R_w} P_{u\geqslant}(K_w) \tag{28}$$

As shown in (28), the expected stock-out cost per year is obtained by multiplying the expected number of trips per year $\left(\frac{1}{R_w}\right)$, the probability of stock-out $(P_{u\geq}(K_w))$ and the cost per stock-out (B_w).

In order to determine the transportation cost from the supplier to the warehouse, we use the same approach that was used for retailer transportation cost. Therefore, we obtain:

$$C_{T_w} = \left(\alpha_s + t_s \left\lceil \frac{\mu_w R_w}{VC} \right\rceil d_{sw}\right) \frac{1}{R_w} \tag{29}$$

where $\frac{\mu_w}{R_w}$ is the replenishment order quantity of the warehouse. Consequently, given (12), (13), (26), (27), (28), (29) and Proposition 1 we obtain:

$$TC_w(R_w) = \frac{A_w}{R_w} + \left(\frac{(R_w + L_{sw})\mu_w}{2} + K_w \sigma_w \sqrt{(R_w + L_{sw})}\right) V_w r_w + \left(\alpha_s + t_s \left\lceil \frac{\mu_w R_w}{VC} \right\rceil d_{sw}\right) \frac{1}{R_w} + \frac{B_w}{R_w} P_{u\geq}(K_w)$$

Let $z = \left\lceil \frac{\mu_w R_w}{VC} \right\rceil$, then, for $R_w = \frac{zVC}{\mu_w}$ we can write $TC_w(R_w)$ as a function of z:

$$TC_w(z) = \frac{A_w \mu_w}{zVC} + \left(\frac{(zVC + L_{sw}\mu_w)}{2} + K_w \sigma_w \sqrt{\left(\frac{zVC}{\mu_w} + L_{sw} \right)} \right) V_w r_w + \frac{(\alpha_s + t_s z d_{sw})\mu_w}{zVC} + \frac{B_w \mu_w}{zVC} P_{u\geq}(K_w) \tag{30}$$

and for $\frac{(z-1)VC}{\mu_w} < R_w < \frac{zVC}{\mu_w}$:

$$TC_w(R_w) = \frac{A_w}{R_w} \left(\frac{(R_w + L_{sw})\mu_w}{2} + K_w \sigma_w \sqrt{(R_w + L_{sw})} \right) V_w r_w + \frac{(\alpha_s + t_s z d_{sw})}{R_w} + \frac{B_w}{R_w} P_{u\geq}(K_w) \tag{31}$$

In Eq. (32) we just need to check integer values of $\frac{\mu_w R_w}{VC}$ to determine the minimum total cost, whereas in (33), the optimal review period at the warehouse can be determined by setting the derivative of the total cost Eq. (33) to zero. Before that we prove following proposition:

Proposition 3. *In Eq. (33), TCw is convex in R_w and is minimized at R_w^*.*

Proof. Given the first derivative with respect to R_w, set = 0:

$$\frac{A_w}{R_w^2} + \frac{(\alpha_s + t_s z d_{sw})}{R_w^2} + \frac{B_w P_{u\geq}(K_w)}{R_w^2} = \frac{\mu_w V_w r_w}{2} + \frac{K_w V_w r_w \sigma_w}{2\sqrt{(R_w + L_{sw})}}$$

Since $\frac{\mu_w V_w r_w}{2} > 0$, always:

$$\frac{[A_w + (\alpha_s + t_s z d_{sw}) + B_w P_{u\geq}(K_w)]}{R_w^2} > \frac{K_w V_w r_w \sigma_w}{2\sqrt{(R_w + L_{sw})}}$$

Dividing both sides by $R_w + L_{sw}$:

$$\frac{2[A_w + (\alpha_s + t_s z d_{sw}) + B_w P_{u\geq}(K_w)]}{R_w^2 (R_w + L_{sw})} > \frac{K_w V_w r_w \sigma_w}{4(R_w + L_{sw})^{1.5}}$$

As a result the 2nd derivative of TC_w is

$$\frac{d^2 TC}{d^2 R_w} = \frac{2[A_w + (\alpha_s + t_s z d_{sw}) + B_w P_{u\geq}(K_w)]}{R_w^2 (R_w + L_{sw})} - \frac{K_w V_w r_w \sigma_w}{4(R_w + L_{sw})^{1.5}} > 0,$$

This implies that the function is convex and results in a minimum value and indicates that R_w^* is:

$$R_w^* = \sqrt{\frac{2[A_w + (\alpha_s + t_s z d_{sw}) + B_w P_{u\geq}(K_w)]}{\left(\mu_w + \frac{K_w \sigma_w}{\sqrt{R_w + L_{sw}}} \right) V_w r_w}} \tag{32}$$

□

The result in (34) does not result in a closed form solution for R_w. We propose an iterative solution to compute R_w. The suggested procedure is to initially set $R_{\text{det}} = \sqrt{\frac{2A_w}{\mu_w V_w r_w}}$ (which is the optimal re-order point of the deterministic case) and then compute R_w^* in each iteration. Because of the convex nature of the total cost function, convergence to the optimal value is ensured if enough iterations are completed.

To find R_w^*, we first check all integer values of $\frac{\mu_w R_w}{VC}$ and determine which one minimizes (32). We start by considering $R_w = 1$, which means the warehouse orders enough to satisfy all demand simultaneously. We determine the number of trucks needed to deliver this amount. This is an upper bound for number of trucks or z. In that case we can analyze all the integer points within that domain. Then we utilize (33) and (34) to determine other possible values (where $\frac{\mu_w R_w}{VC}$ is not integer). This procedure can be outlined as follows:

Algorithm 2.

Part 1: Compute R_w for integer points.
Step 1. Set $R_w = 1$ and calculate the domain of z.
Step 2. Compute $TC_w(z)$ using Eq. (30).
Step 3. While $z > 0 z = z - 1$ and go to step 2.
Step 4. Set TC_{int} as the minimum of all computed $TC_w(z)$ in Step 2.
Part 2: Compute R_w for non-integer points.
Step 5. Initialization Set $i = 1$ and $R_w^0 = \sqrt{\frac{2A_w}{\mu_w V_w r_w}}$.
Step 6. Compute R_w^i and $TC_w(R_w^i)$ using $R_w^i = \sqrt{\frac{2[A_w + (\alpha_s + t_s z d_{sw}) + B_w P_{u\geq}(K_w)]}{\left(\mu_w + \frac{K_w \sigma_w}{\sqrt{R_w^{i-1} + L_{sw}}} \right) V_w r_w}}$ and Eq. (31).

Step 7. Iteration While $|TC_w(R_w^{i-1}) - TC_w(R_w^i)| > \varepsilon$, $i = i + 1$ and go to 6.
Part 3: Comparison.
Step 8. Compare the values obtained in Steps 4 and 7 and return the smallest $TC_w(R_w)$ and the corresponding R_w^*.
Step 9. End

5. Centralized ordering (collective) optimization

We make a number of different assumptions in the centralized ordering model. In the decentralized ordering, it is assumed that each retailer finds its optimal order quantity, sends it to the warehouse, and receives it afterward. The main motivation of the centralized ordering policy is to explore whether such a policy leads to a lower total system-wide cost by improving the inventory and transportation decisions.

We propose a collective form of ordering by retailers and plan to minimize the inventory cost of the retailers and the warehouse jointly. The warehouse observes a sequence of demands from a group of retailers positioned in a given region. Ideally, these demands should be shipped immediately. Retailers observe their customers' demand and then collaborate to explore the optimal joint order amount and send it to the warehouse. We formulate a continuous review model for the centralized scenario to find the optimal re-order point, S, and optimal order quantity, Q, that minimize overall system costs. Based on this introduction, the retailers decide the re-order point and the warehouse determines the replenishment quantity.

We also assume the use of a similar forecasting technique and inventory policy that we used in the former model. We set the objective function of the system under consideration as:

$$\text{Minimize}\, Z^c \left(= \sum_{j=1}^{N} TC_j(Q_j) + TC_w \right)$$

Therefore, the warehouse and the retailers must optimize their decision variables in a way to reduce the total cost of the system. This means that we first find the total cost of the system (which is the summation of the warehouse and retailers costs) and then try to determine the optimal value for the joint order size.

Each retailer's costs include transportation cost, cost of replenishment and carrying cost, and cost of stock-out. We adopt the model of Daganzo (2005) for one-to-many distribution model transportation cost and Burns et al. (1985) for distribution strategy to minimize transportation cost, and we develop a formula to include the impact of the transportation cost in our model. First we assume a vehicle is routed in a way to minimize total distance traveled, which is the main factor in our model. We make a clustering of the retailers in a region (Fig. 2). Therefore, it is assumed that we want to solve a national (regio-

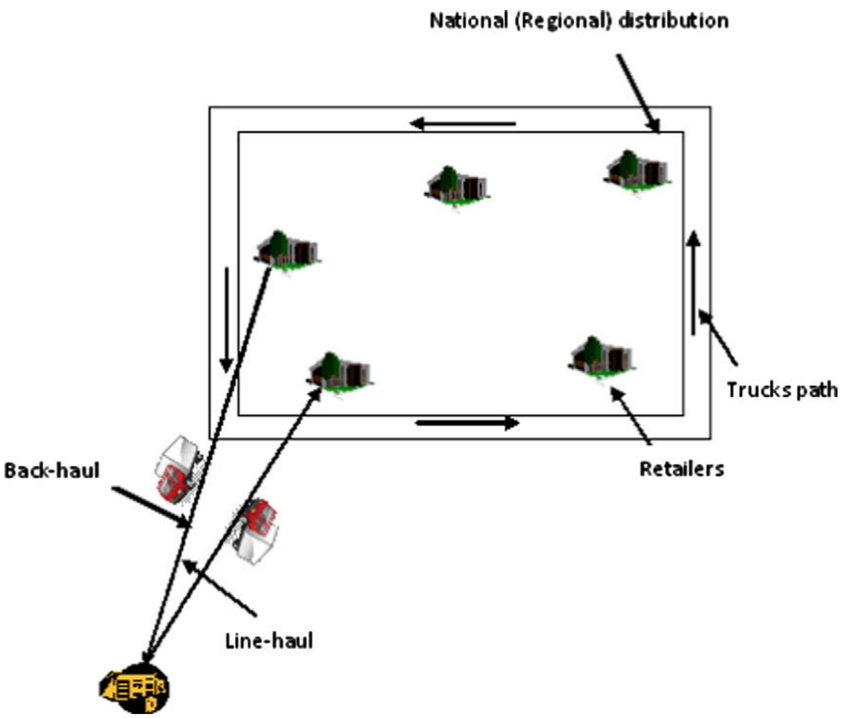

Fig. 2. Clustering of the Retailers.

nal) distribution problem. Then all the vehicles are considered for a combined tour. In this case, the total distance to serve all the retailers in the given region includes three elements: (1) Line-Haul (trip from warehouse to the first retailer in the region or entrance of the region); (2) Back-Haul (trip from last retailer to the warehouse); and (3) local (trips from the first retailer to the last one in the region). In our model we only focus on Line-Haul and neglect Back-Haul as we have done for decentralized ordering model.

Several methodologies have been applied to obtain vehicle routes from origin(s) to destination(s) at a minimum cost such as Shortest Path Problem, Minimum Spanning Tree, Vehicle Routing Problem, Traveling Salesman Problem, etc. In this research we try to estimate the total distance to serve all the retailers or the expected distance travel of the truck(s) and neglect the distribution methodologies and the routing problems. Therefore, given the assumptions mentioned before, we have:

$$\text{Expected total distance traveled}(d_{wR}) = \text{Line-Haul distance} + \text{local distance} \tag{33}$$

To compute the Line-Haul distance, we suppose retailer i is the first retailer visited in the region and we label the distance between this retailer and the warehouse as d_{wi}. Therefore, we have:

$$\text{Yearly Line-Haul cost of transport} = \left(\alpha_w + t_w d_{wi}\left\lceil\frac{Q_R}{VC}\right\rceil\right)\frac{\sum_{j=1}^{N} D_j}{Q_R} \tag{34}$$

In the above equation, Q_R indicates the total combined-order quantity of all the retailers. Now, we should compute local distance. Given the analysis of the shortest Euclidean path connecting m customers located randomly in a delivery region, our local distance can be approximated by (see Burns et al., 1985 and Daganzo 2005)

$$\text{Local distance} = k\sqrt{\frac{mN}{\rho}}$$

where N is the number of the retailers and m indicates the maximum number of the stops made by a truck (vehicle), ρ indicates the retailer density (retailers per square kilometer) and k is a constant value. However, due to the fact that more than one truck might need to carry the goods, the average of local distance per year is

$$\left(\frac{k\sqrt{\frac{mN}{\rho}}}{\left\lceil\frac{Q_R}{VC}\right\rceil}\right)\frac{\sum\limits_{j=1}^{N} D_j}{Q_R} \tag{35}$$

We now have:

$$\text{Total cost of transport} = \left(\alpha_w + t_w d_{wi}\left\lceil\frac{Q_R}{VC}\right\rceil + \frac{k\sqrt{\frac{mN}{\rho}}}{\left\lceil\frac{Q_R}{VC}\right\rceil}\right)\frac{\sum_{j=1}^{N} D_j}{Q_R} \tag{36}$$

Assuming $m = N$ yields an upper bound in our problem. This assumption is discussed later in Section 6.2.1. Hence, the total combined cost of transport is:

$$C_{T_R} = \left(\alpha_w + t_w d_{wi}\left\lceil\frac{Q_R}{VC}\right\rceil + \frac{kN\sqrt{\frac{1}{\rho}}}{\left\lceil\frac{Q_R}{VC}\right\rceil}\right)\frac{\sum_{j=1}^{N} D_j}{Q_R} \tag{37}$$

We stated that only one item is being distributed in our model. Thus, the cost of replenishment will be equally distributed among the retailers. The combined cost of replenishment is given by:

$$C_{R_R} = \frac{A_R \sum_{j=1}^{N} D_j}{Q_R} \tag{38}$$

Moreover, we assume that the retailers decide the re-order point. However, carrying costs might be different in each retailer. From the decentralized ordering model, we found the carrying cost for each retailer as, $C_{C_j} = \left(\frac{Q_j}{2} + K_j \sigma_{jL_{wj}}\right) V_r r_j$. Here we propose an allocation rule based on the ratio between each retailer demand to the total demand and the retailers as $Q_j = \frac{Q_R D_j}{\sum_{j=1}^{N} D_j}$, hence, the combined carrying cost is as follows:

$$C_{C_R} = \left(\frac{Q_R}{2} + K_R \sigma_{RL_{wR}}\right)\sum_{j=1}^{N} V_j r_j \tag{39}$$

In the preceding equation, K_R indicates the safety factor of the retailers and is determined by the retailers for both the warehouse and retailers. L_{wR} indicates the average lead-time from warehouse to the retailers in the region (for convenience we assume that this value is fixed and depends on the Line-Haul and local distances). Ultimately, in centralized ordering $\mu_{RL_{wR}}$ and $\sigma_{RL_{wj}}$ can be determined by:

$$\mu_{RL_{wR}} = L_{wR} \sum_{j=1}^{N} \mu_j \tag{40}$$

and

$$\sigma_{RL_{wR}} = \sqrt{L_{wR} \sum_{j=1}^{N} \sigma_j^2} \tag{41}$$

Consequently, the total cost of the retailers is:

$$TC_R(Q_R, K_R) = \frac{A_R \sum_{j=1}^{N} D_j}{Q_R} + \left(\frac{Q_R}{2} + K_R \sigma_{RL_{wR}}\right) \sum_{j=1}^{N} V_j r_j + \left(\alpha_w + t_w d_{wi} \left\lceil \frac{Q_R}{VC} \right\rceil + \frac{kN\sqrt{\frac{1}{\rho}}}{\left\lceil \frac{Q_R}{VC} \right\rceil}\right) \frac{\sum_{j=1}^{N} D_j}{Q_R} \tag{42}$$

In the same way, the cost of the warehouse is determined as follows:

$$TC_w(Q_R, K_R) = \frac{A_w \sum_{j=1}^{N} D_j}{Q_R} + \left(\frac{Q_R}{2} + K_R \sigma_{wL_{sw}}\right) V_w r_w + \left(\alpha_s + t_s d_{sw} \left\lceil \frac{Q_R}{VC} \right\rceil\right) \frac{\sum_{j=1}^{N} D_j}{Q_R} + \frac{B_w P_{u\geq}(K_R) \sum_{j=1}^{N} D_j}{Q_R} \tag{43}$$

Recall that we assume that the retailers decide the re-order point and the warehouse determines the replenishment quantity. We try to determine the optimal values for this situation in the following section.

5.1. Joint optimization

The goal of centralized ordering is to jointly minimize the combined inventory cost of retailers and the warehouse. Given (41) and (42), the total cost is

$$\begin{aligned} TC(Q_R, K_R) = {} & \frac{(A_R + A_w)\sum_{j=1}^{N} D_j}{Q_R} + \frac{Q_R}{2}\left(\sum_{j=1}^{N} V_j r_j + V_w r_w\right) + K_R\left(\sigma_{RL_{wR}} \sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right) \\ & + \left((\alpha_w + \alpha_s) + (t_w d_{wi} + t_s d_{sw})\varphi + \frac{kN\sqrt{\frac{1}{\rho}}}{\varphi}\right) \frac{\sum_{j=1}^{N} D_j}{Q_R} + \frac{B_w P_{u\geq}(K_R) \sum_{j=1}^{N} D_j}{Q_R} \end{aligned} \tag{44}$$

where $\varphi = \lceil \frac{Q_R}{VC} \rceil$.

Proposition 4. $TC(Q_R, K_R)$ is convex in K_R.

Proof. Since the derivative of cumulative distribution function is probability density function, we get

$$\frac{dP_{u\geq}(K_R)}{dK_R} = -f(K_R)$$

As a result

$$\frac{dTC(Q_R, K_R)}{dK_R} = \left(\sigma_{RL_{wR}} \sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right) - \frac{B_w \sum_{j=1}^{N} D_j}{Q_R} f(K_R) = 0$$

Given the density function of the standard normal distribution, we have,

$$\frac{dTC(Q_R, K_R)}{dK_R} = \left(\sigma_{RL_{wR}} \sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right) - \frac{B_w \sum_{j=1}^{N} D_j}{Q_R} \left(\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{K_R^2}{2}\right)\right) = 0$$

Finally,

$$\frac{d^2 TC(Q_R, K_R)}{dK_R} = \frac{B_w \sum_{j=1}^{N} D_j}{Q_R} \left(\frac{2K_R}{\sqrt{2\pi}} \exp\left(-\frac{K_R^2}{2}\right)\right) \geq 0$$

From the previous discussion, we have two scenarios. For $Q_R = \varphi VC$ we can write $TC(Q_R, K_R)$ as:

$$\begin{aligned} TC(Q_R, K_R) = {} & \frac{(A_R + A_w)\sum_{j=1}^{N} D_j}{\varphi VC} + \frac{\varphi VC}{2}\left(\sum_{j=1}^{N} V_j r_j + V_w r_w\right) + K_R\left(\sigma_{RL_{wR}} \sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right) \\ & + \left((\alpha_w + \alpha_s) + (t_w d_{wi} + t_s d_{sw})\varphi + \frac{kN\sqrt{\frac{1}{\rho}}}{\varphi}\right) \frac{\sum_{j=1}^{N} D_j}{\varphi VC} + \frac{B_w P_{u\geq}(K_R) \sum_{j=1}^{N} D_j}{\varphi VC} \end{aligned} \tag{45}$$

□

Table 1
Decentralized ordering – retailers optimal quantities and optimal cost.

	μ_j	σ_j	L_{wj}	$\sigma_{jL_{wj}}$	V_j	r_j	A_j	d_{wj}	P_j	K_j	Q_j^*	C_{R_j}	C_{C_j}	C_{T_j}	$TC_j(Q_j^*)$
Retailer 1	857	15	.04	2.9	90	1.0	100	15	0.95	1.64	90	952.2	4485.5	3094.7	8532.4
Retailer 2	698	12	.04	2.3	90	1.0	100	25	0.9	1.28	94	742.6	4501.4	3527.1	8771.1
Retailer 3	983	24	.08	6.7	90	1.0	100	20	0.99	2.33	100	983.0	5921.3	3932.0	10836.3
Retailer 4	687	6	.05	1.3	90	1.0	100	25	0.95	1.64	94	730.9	4424.6	3471.5	8627.0
Retailer 5	786	13	.11	4.3	90	1.0	100	28	0.90	1.28	100	789.0	5001.8	4102.8	9893.6
Retailer 6	921	21	.09	6.4	90	1.0	100	18	0.90	1.28	98	939.8	5144.7	3477.2	9561.7
Total cost												5137.4	29,479.3	21,605.4	56,222.1

For $(\varphi - 1)VC < Q_R < \varphi VC$, following proposition can be satisfied:

Proposition 5. *For $(\varphi - 1)VC < Q_R < \varphi VC, TC(Q_R, K_R)$ is convex in K_R, Q_R and jointly.*

Proof. We earlier proved that $TC(Q_R, K_R)$ is convex in K_R in Proposition 4. For Q_R we get:

$$\frac{d^2TC(Q_R,K_R)}{dQ_R} = \frac{2(A_R + A_w)\sum_{j=1}^{N} D_j}{Q_R^3} + 2\left((\alpha_w + \alpha_s) + (t_w d_{wi} + t_s d_{sw})\varphi + \frac{kN\sqrt{\frac{1}{\rho}}}{\varphi}\right)\frac{\sum_{j=1}^{N} D_j}{Q_R^3} + \frac{2B_w P_{u\geq}(K_R)\sum_{j=1}^{N} D_j}{Q_R^3} \geq 0$$

Now need to prove the jointly convexity. For this purpose we need to build Hessian matrix.

It is straightforward to show that following relation is satisfied:

$$\begin{vmatrix} \frac{d^2TC(Q_R,K_R)}{dK_R} & \frac{d^2TC(Q_R,K_R)}{dQ_R dK_R} \\ \frac{d^2TC(Q_R,K_R)}{dQ_R dK_R} & \frac{d^2TC(Q_R,K_R)}{dQ_R} \end{vmatrix} \geq 0$$

The above propositions also result in following equations for K_R and Q_R:

$$K_R = \sqrt{2\ln\left(\frac{B_w\sum_{j=1}^{N} D_j}{\sqrt{2\pi}Q_R\left(\sigma_{RL_{wR}}\sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right)}\right)} \tag{46}$$

and

$$Q_R^* = \sqrt{\frac{2\sum_{j=1}^{N} D_j\left((A_R + A_w) + (\alpha_w + \alpha_s) + (t_w d_{wi} + t_s d_{sw})\varphi + \frac{kN\sqrt{\frac{1}{\rho}}}{\varphi} + B_w P_{u\geq}(K_R)\right)}{\left(\sum_{j=1}^{N} V_j r_j + V_w r_w\right)}} \tag{47}$$

□

Now we apply following procedure to determine the solution for the centralized model.

Algorithm 3.

Part 1: Integer points
Set $Q_R = \sum_{j=1}^{N} D_j$ and calculate domain of φ .Set $\varphi \in (0, \frac{\sum_{j=1}^{N} D_j}{VC}]$
Step 1.Set $Q_j = \varphi.VC$ and compute $h = \frac{B_w\sum_{j=1}^{N} D_j}{\sqrt{2\pi}Q_R\left(\sigma_{RL_{wR}}\sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right)}$. If h > 1 then compute K_j by using Eq. (47) else $K_j = 0$.
Step 2. Compute $TC(Q_R, K_R)$ using Eq.(46).
Step 3. While $\varphi > 0, \varphi = \varphi - 1$, go to step 1.
Step 4. Set $TC_{\text{int}}(Q_R, K_R)$ as the minimum of all computed $TC(Q_R, K_R)$ in Step 2.
Part 2: non-integer points.
Step 5. Initialization Set $i = 1$ and $Q^0 = EOQ = \sqrt{\frac{2(A_R + A_w)\sum_{j=1}^{N} D_j}{\left(\sum_{j=1}^{N} V_j r_j + V_w r_w\right)}}$.
Step 6. Compute h. If $h > 1$ then $K^i = \sqrt{2\ln\left(\frac{B_w\sum_{j=1}^{N} D_j}{\sqrt{2\pi}Q^{i-1}\left(\sigma_{RL_{wR}}\sum_{j=1}^{N} V_j r_j + \sigma_{wL_{sw}} V_w r_w\right)}\right)}$ else $K_j = 0$.
Step 7. Compute Q_R and $TC(Q_R, K_R)$ using $Q^i = \sqrt{\frac{2\sum_{j=1}^{N} D_j\left((A_R + A_w) + (\alpha_w + \alpha_s) + (t_w d_{wi} + t_s d_{sw})\varphi + \frac{kN\sqrt{\frac{1}{\rho}}}{\varphi} + B_w P_{u\geq}(K^i)\right)}{\left(\sum_{j=1}^{N} V_j r_j + V_w r_w\right)}}$ and Eq.(45).

Step 8. Iteration While $|TC^i(Q_R, K_R) - TC^{i-1}(Q_R, K_R)| > \varepsilon$, $i = i + 1$ and go to 6.
Part 3: Comparison.
Step 9. Compare the values obtained in Step 4 and Step 8 and return the smallest $TC(Q_R, K_R)$ and the corresponding Q_R^* and K_R^*.

6. Numerical analysis

In order to analyze the discussed model we consider a numerical example. The objective of this section is twofold. First, we want to calculate the defined costs of each model and calculate the optimal order quantity of each retailer, and the second, we want to compare the total costs with different scenarios. The number of the retailers, N, is assumed to be *six* and *one* warehouse is considered. For the decentralized ordering case, input data and resulting inventory costs are shown in Tables 1 and 2, respectively, for the retailers and the warehouse. In this case, we set $VC = 100$, $t_s = t_w = \$15$ and $\alpha_s = \alpha_w = \$100$. The results are summarized in Tables 1–3, respectively, for decentralized and centralized model. We implemented the procedures in MATLAB 7.4.0.

Using the results of Table 1 and 2, we see that the total cost of the model for the decentralized ordering is about \$97,030. The proportion of total cost for each individual retailer attributable to transportation to the warehouse is 36%, 40%, 36%, 40%, 41% and 36%.The warehouse transportation cost is high as well and contains 38% of the total cost. Finally, Table 1 shows that contrary to what some inventory managers believe, in certain circumstances, full truck loading is not always cost-effective (we discuss this case more in Section 6.2.)

To establish the value of our approach consider the retailer 1 in Table 1 where $D = 857$ units, $A = 100$, $h = 90$. Using the classical EOQ we find $EOQ = 44$ and the total cost roughly \$3928, whereas we found $Q_1^* = 90$ and $TC_1 = \$8603$. Now suppose that we consider $Q^* = 44$ of classical EOQ as the optimal order quantity then the total cost, including transportation cost, is equal \$10,693, which is 25% above the total cost that we determined in the first retailer.

Table 3 illustrates the solution for the centralized model. The total cost of the centralized ordering model is \$48,180. The main observation from Table 3 can be the fact that transportation contributes only 2.6% of the total cost and, in contrast, the carrying and replenishment costs contribute much more to the total cost, with 81% and 15%, respectively.

To build a better understanding of the performance differences, we make Table 4 to compare the total cost of the decentralized ordering model with the centralized ordering model and display the improvements. As shown, the effects of using the centralized ordering model in our inventory model immediately become noticeable; the difference between the total costs of the decentralized and centralized ordering model is over \$48,000 which is over 50% of the decentralized ordering cost.

The centralized ordering model also results in a significant costs savings in transportation cost which is over \$38,000. A noticeable cost saving is achieved in carrying cost which is over \$18,000. Since most of the cost reduction is achieved in the transportation cost and carrying cost and both of these costs contain a high percentage of the total cost in our model, making a collective ordering among the retailers can considerably reduce the total cost.

We also observe a slight increase in replenishment cost and shortage cost of the centralized ordering model. This is mainly because of the fact that warehouse order size quantity ($=R_w^* \cdot \mu_w = .0405 * 4935 \approx 200$) in the decentralized ordering model is greater than the Q_R^* centralized case.

Table 2
Decentralized ordering – warehouse optimal review period and optimal cost.

	μ_w	σ_w	V_w	r_w	K_w	d_{sw}	B_w	P_w	R_w^*	C_{R_w}	C_{C_w}	C_{T_w}	C_{s_w}	$TC_w(R_w^*)$
Warehouse	4935	91	60	1	1.6	20	150	0.95	0.0405	1974.0	20,877.0	17273.0	202.8	40,326.8

Table 3
Centralized ordering.

d_{wi}	k	ρ	Q_R^*	K_R^*	C_{R_w}	C_{C_w}	C_{T_w}	C_{s_w}	$TC(Q_R^*, K_R^*)$
15	.6	0.1	100	1.1592	8883.0	31,748.0	736.4	911.9	42,279.3

Table 4
Cost comparison.

	C_{R_w}	C_{C_w}	C_{T_w}	C_{s_w}	Total cost
Retailer costs	5137.4	29,479.3	21,605.4	–	56,222.1
Warehouse costs	1974	20,877	17,273	202.8259	40,326.8
Total Decentralized ordering costs	7111.4	50356.3	38878.4	202.8259	96,548.9
Centralized ordering costs	8883	31,748	736.3842	911.9496	42,279.3

The most salient conclusion that can be drawn from these results is that the centralized (collective) optimization has lower cost and so better performance compared to decentralized optimization, and few parameters are in fact impacting the total costs of inventory including the transportation costs. However, we should note that the magnitude of savings is also highly dependent on numerical values and parameters which are used to solve the models. In the next section we utilize a sensitivity analysis for further investigation.

6.1. Sensitivity analysis

Our numerical example illustrates the sensitivity of the solution relative to the model parameters. In this section a few scenarios have been chosen to examine the effect of our assumptions on the total cost of the model. We also clarify a few of our observations from the analysis.

6.1.1. Change in ρ

In the first scenario we examine different values for the location density parameter of the retailers. In this case the total cost of decentralized ordering remains unchanged. The result is represented in Fig. 3. From the result we realized that for smaller ρ the total cost of the centralized ordering is larger. This implies that when retailers are more concentrated, centralized ordering model has even higher efficiency.

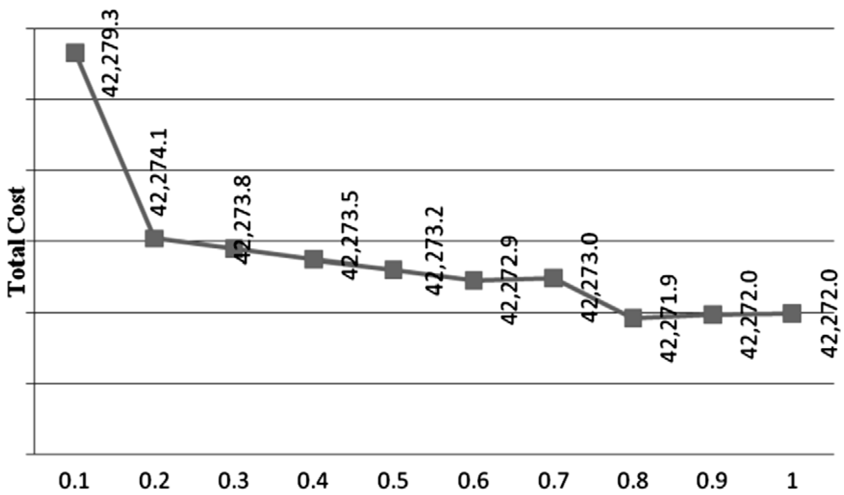

Fig. 3. Change in retailer density ρ.

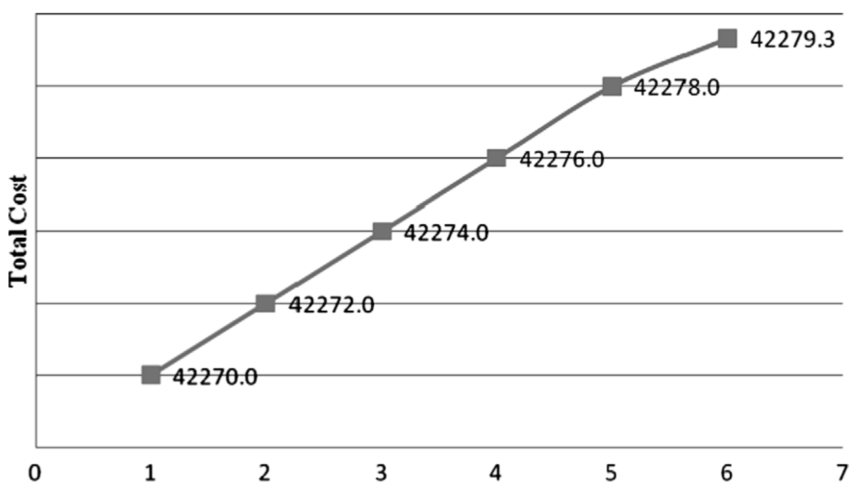

Fig. 4. Change in number of visited retailers N.

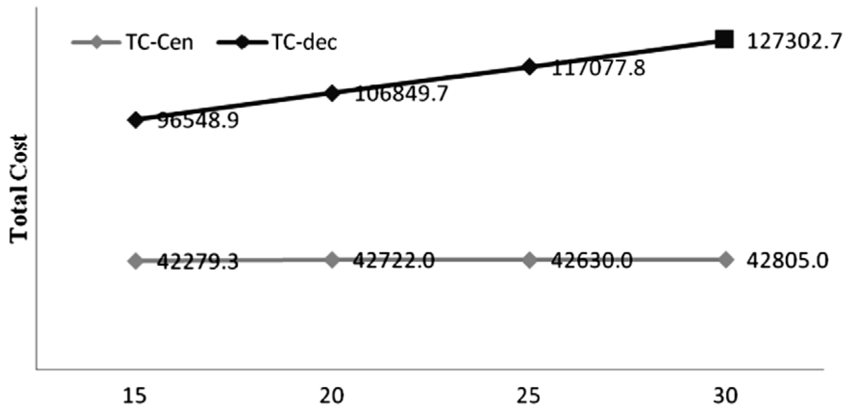

Fig. 5. Change in variable cost of transportation.

6.1.2. Change in N

Recall that in the centralized ordering model we assumed that every retailer is visited in every trip or $m = N$. Here we observe different scenarios when $N - 1$, $N - 2$, ..., 1 of the retailers are visited, the resulting costs shown in Fig. 4. We observe a slight reduction in the total cost from $N = 6$ to $N = 1$.

6.1.3. Change in variable cost of transportation

In the third scenario, we vary the price for the variable cost of transport from warehouse to retailers and supplier to warehouse. We move the value of this cost from $15 to $30 in increments of five units. The results obtained here still are in favor of centralized ordering and show that as much as we increase the full load cost of transport, the difference between costs of decentralized ordering and centralized ordering will grow (see Fig. 5).

6.1.4. Change in carrying cost

In the last scenario, we vary the carrying cost of the warehouse and retailers to find out the effects of that on our policy. We keep the ratio of the carrying cost of the warehouse to the retailers at a constant rate of 2/3; for example, if the carrying cost of the retailers is 120 then carrying cost of the warehouse must be 80. This is extended for more carrying cost with increments of 30 units and should be taken into consideration when reading the Fig. 6.

Given Fig. 6, the most evident conclusion is that for higher retailer and warehouse carrying costs, the centralized ordering model still has better cost value.

6.2. Key observations

Observation 1. For low variable transportation costs or for short distances, it is not economical to use full truckloads. However, for large distances or high variable transportation costs it is always preferred to order full truckloads.

Observation 2. In the decentralized ordering model, full truckloads are less preferred as the carrying cost at the retailers increase.

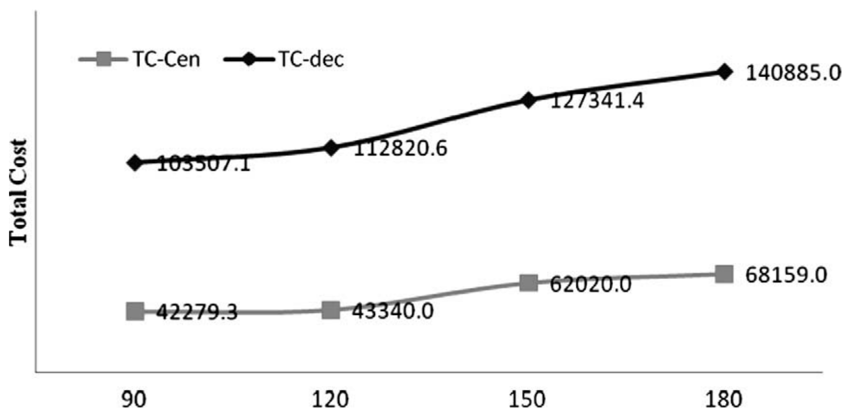

Fig. 6. Change in carrying cost.

Observation 3. In the decentralized ordering model as the carrying cost at the warehouse increases the corresponding review period decreases.

Observation 4. In the centralized ordering model, full truckloads are less preferred as the carrying cost at the retailers and the warehouse increase.

Observation 5. In the centralized ordering model, as the carrying cost at the retailers and the warehouse increase, the corresponding order quantity and safety factor decrease.

7. Conclusions

We have formulated a multi-level inventory model that includes transportation costs for planning the replenishment of a single commodity. The contributions of the paper to the literature are threefold. First, to extend traditional Economic Order Quantity model in order to minimize the total inventory cost while considering a discrete transportation cost. Second, determining the optimal strategy of the warehouse to decide how often to place orders. Finally, to develop a collective form of ordering by retailers and plan to minimize the inventory cost of the retailers and the warehouse jointly.

We developed two models considering the scenarios of centralized ordering and decentralized ordering. A numerical example was solved for both models, with some sensitivity analysis for the centralized ordering scenario. Results indicate that having collaboration among the retailers and the warehouse or applying a collective ordering strategy results in reduced costs when compared to the decentralized ordering strategy. A remarkable result that was achieved by the numerical example indicates that utilizing full truckloads is not always optimal. Furthermore, it was shown that the transportation cost contains a considerable percentage of the total cost, while this cost has been usually overlooked.

The models presented here can be extended to include a multi-item multi-level inventory model or true costs of transportation, like environmental costs, or costs of return flow due to lack of demand (excess inventory) or customer returns. A further extension could be obtaining a solution that shares the benefits of the collaboration.

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