International Journal of Production Research

ISSN: 0020-7543 (Print) 1366-588X (Online) Journal homepage: https://www.tandfonline.com/loi/tprs20

Two-stage hybrid flow shop batching and lot streaming with variable sublots and sequence-dependent setups

Shasha Wang, Mary Kurz, Scott Jennings Mason & Eghbal Rashidi

To cite this article: Shasha Wang, Mary Kurz, Scott Jennings Mason & Eghbal Rashidi (2019) Two-stage hybrid flow shop batching and lot streaming with variable sublots and sequence-dependent setups, International Journal of Production Research, 57:22, 6893-6907, DOI: 10.1080/00207543.2019.1571251

To link to this article: https://doi.org/10.1080/00207543.2019.1571251

Published online: 12 Feb 2019.

International Journal of Production Research, 2019
Vol. 57, No. 22, 6893–6907, https://doi.org/10.1080/00207543.2019.1571251

Two-stage hybrid flow shop batching and lot streaming with variable sublots and sequence-dependent setups

Shasha Wang [a*], Mary Kurz[a], Scott Jennings Mason[a] and Eghbal Rashidi[b]

[a] *Department of Industrial Engineering, Clemson University, Clemson, SC, USA;* [b] *Leavey School of Business, Santa Clara University, Santa Clara, CA, USA*

(*Received 18 January 2018; accepted 9 January 2019*)

A paint manufacturing firm's customers typically place orders for two or more products simultaneously. Each product belongs to a family that denotes batching compatibility during manufacturing. Further, products can be split into several sublots to allow overlapping production in a two-stage hybrid flow shop wherein various identical, capacitated machines operate in parallel at each stage. We present a mixed-integer linear program (MILP) for this integrated batching and lot streaming problem with variable sublots, incompatible job families, and sequence-dependent setup times. The model determines the number and size of sublots for each product and the production sequencing for each sublot such that the total weighted completion time is minimised. To promote practical implementation, we develop and evaluate heuristics to efficiently solve this problem.

Keywords: Batch scheduling; lot streaming; sequence-dependent setups; incompatible product family; hybrid flow shop

1. Introduction

As manufacturing enterprises continue to endure market, environment, and energy usage pressures (Wang et al. 2019), improving customer satisfaction, decreasing production cycle time, and reducing costs remain key factors for successful businesses. Lot streaming has emerged as an attractive method for reducing makespan, cycle time, average work-in-process inventory, required storage space, and material handling equipment requirements. Lot streaming splits a batch of jobs into several sublots which can be processed in an overlapping fashion on successive stages in a multi-stage manufacturing environment. Usually, lot streaming focuses on determining the number and size of sublots for a product and the assignment and sequence of these sublots on machines to optimise performance criteria while satisfying required constraints.

Batching is another method used to handle production lots in scheduling. Batch scheduling focuses on finding capacity-feasible schedules that optimise given objective function(s) while meeting required constraints. The intent of utilising batching and lot streaming is to minimise time-related objectives such as makespan. However, batching and lot streaming use different approaches. Batch scheduling requires grouping products to form a batch and then sequencing batches on machines. Typically, batches cannot be split during manufacturing (Yin et al. 2016), resulting in increased machine utilisation, reduced setup times and lower completion times. Batch processing is typically used in semiconductor wafer fabrication facilities (Shahvari and Logendran 2018; Mönch and Roob 2017). In lot streaming problems, batches are usually assumed to be given (Mukherjee, Sarin, and Singh 2017; Zhang et al. 2017) and the decision is only when and how to split batches. Sublots can be processed in an overlapping way on successive stages so that makespan/completion time can be minimised. Lot streaming should only be used when the entire volume of a batch is greater than the machines' capacity. Our work focuses on this aspect of lot streaming.

This study is motivated by the first author's work experience at a coating company. Customers place orders for one of two product groups. One product group consists of primer and top coat paint; the other is composed of primer, top coat paint, and undercoat paint. Any paint order can be divided into hundreds of subcategories, each associated with an incompatible product family. Sequence-dependent setup times occur when production switches between product families. Figure 1 describes the basic production steps required to manufacture each coating system component. Please see Olson and Schniederjans (2000) and Adonyi et al. (2008) for more details about the paint industry. The manufacturing environment is a two-stage hybrid flow shop (2-HFS), a flow shop with multiple stages where, in at least one stage, multiple parallel, identical machines exist (Kurz and Askin 2004; Zandieh and Rashidi 2009; Rashidi, Jahandar, and Zandieh 2010). Each

*Corresponding author. Email: shashaw@g.clemson.edu

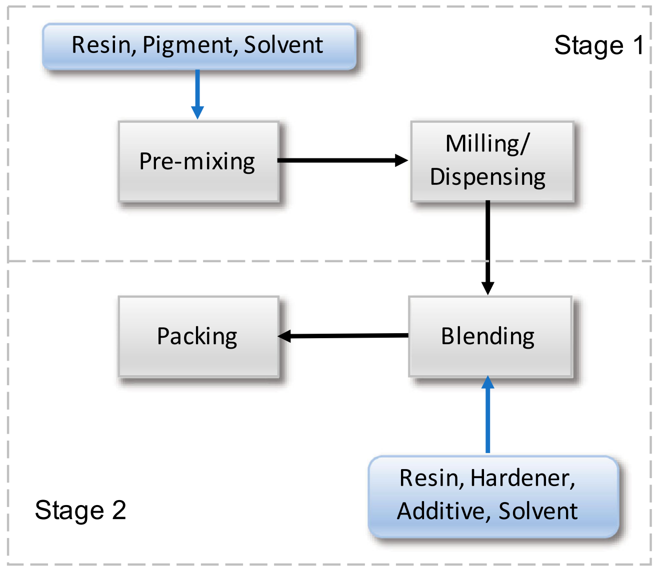


Figure 1. Paint production process.

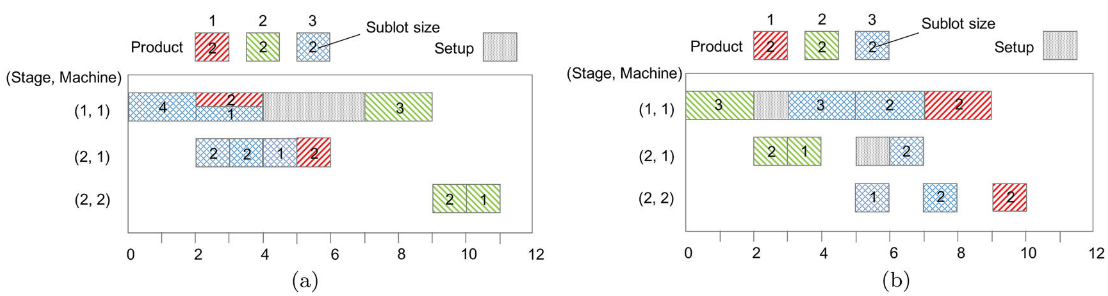


Figure 2. Optimal solution of the example. (a) Integrated batching with lot streaming and (b) Lot streaming.

batch is processed by exactly one machine at each stage. If the demand for a product is larger than machine capacity, it must be divided into several sublots. Each sublot is considered as a batch. If a sublot is smaller than machine capacity, sublots of other products (belonging to the same product family) can be manufactured in this batch to up to machine capacity.

For example, consider three orders wherein each order contains one product. The demand (product family) of each product is 2 (1), 3 (2), 5 (3). There is one machine (capacity of 4) in stage 1 and two machines (capacity of 2) in stage 2. Figure 2(a) shows the Gantt chart of the optimal solution for this example which allows batching and lot streaming simultaneously. The total weighted completion time (TWCT) of all sublots is 84. Product 3 (product 2) is split into 2 (1) sublots in stage 1, and 3 (2) sublots in stage 2. Since products 1 and 3 belong to the same product family, one sublot of product 1 and one sublot of product 3 are processed in the same batch position (2nd) on stage 1's single machine simultaneously. Sublot sizes vary across the two stages, given the relationship between product 3's demand of 5 and the machine capacity in each stage. When considering lot streaming alone (Figure 2(b)), the TWCT is 87, illustrating that scheduling using integrated batching and lot streaming can improve upon using lot streaming alone.

In this study, a set of approaches is proposed for 2-HFS scheduling problems which consider batching and lot streaming simultaneously. The main contributions of this paper are threefold:

(1) An MILP model that incorporates batching and lot streaming is proposed to determine sublot sizes and sequences for multiple products in a 2-HFS to minimise total weighted completion time while satisfying customer demand.
(2) An effective lower bound (LB) is created to evaluate the performance of proposed algorithms.
(3) Heuristics are developed mixing three approaches to sequence products and three methods for splitting products. Experimental analyses confirm the recommended heuristic's ability to help companies make effective production scheduling decisions.

The rest of the paper is organised as follows. After the literature is reviewed in Section 2, detailed mathematical models and a LB are presented in Section 3. Next, heuristics are presented in Section 4. Computational experiments are conducted in Section 5, and conclusions are offered in Section 6.

2. Literature review

A recent review of lot streaming in the literature can be found in Cheng, Mukherjee, and Sarin (2013). The authers review lot streaming problems for time-based and cost-based objective functions. Machine environments such as flow shops, parallel machines, hybrid flow shops, job shops, open shops, and two-stage assembly systems are discussed. Trietsch and Baker (1993) provide basic models and algorithms for the lot streaming problem and complexity classifications for some lot streaming problems.

Feldmann and Biskup (2008) categorize lot streaming problems according to machine configuration, product type, sublot type, and other criteria. Equal sublots refer to the case wherein the size of all sublots is fixed and equal for all products. Problems with consistent sublots allow each product to have its own sublot size that remains constant for all stages/processes. Variable sublots cases contain no restrictions on sublot sizes across machines; sublot sizes may change when transferring between machines.

Biskup and Feldmann (2006) conclude that variable sublots can lead to large improvements in makespan for flow shops compared to other sublot types. The authors claim that they develop the first MILP formulation for lot streaming with variable sublots in m-machine flow shops. Defersha and Chen (2010) extend this model to multiple products and develop a hybrid genetic algorithm to improve computational efficiency. Defersha and Chen (2012) propose a parallel genetic algorithm for the lot streaming problem in a hybrid flexible flow shop with sequence-dependent setup times, release times for machines, and machine eligibility constraints. Pan et al. (2011) develop a discrete artificial bee colony algorithm for the lot streaming flow shop scheduling problem to minimise total weighted earliness and tardiness. Multi-objective lot streaming problems in blocking flow shops with interval processing time are studied by Han et al. (2016). Bożek and Werner (2017) propose a two-stage optimisation procedure for a flexible job shop scheduling problem with lot streaming and lot sizing of variable sublots. The makespan is minimised in the first stage and the sizes of sublots are maximised in the second stage.

Potts and Kovalyov (2000) provide a detailed review on algorithms of batch scheduling problems. Erramilli and Mason (2006) study a multiple orders per job batch single machine scheduling problem with compatible job families wherein jobs that belong to any family may be grouped to form a production batch to minimise the total weighted tardiness. They propose simulated annealing based-heuristics which are found to produce near-optimal solutions in seconds. Erramilli and Mason (2008) consider the same problem with incompatible job families in which only jobs from the same family can be batched together. Heuristic methods are developed to minimise total weighed tardiness. Batch scheduling with sequence-independent and sequence-dependent setup times in flow shops are studied by Pranzo (2004) and Logendran, deSzoeke, and Barnard (2006). Lin and Liao (2012) study a batch scheduling problem in a two-stage assembly shop to minimise the weighted sum of makespan, total completion time, and total tardiness. A full batch family sorting heuristic combined with rolling horizon scheduling strategy is developed for medium- and large-size problems to minimise total weighted makespan, total completion time, and total tardiness. Real-life case studies show that their algorithm is much better than the current method used by the studied company. Shi and You (2016) propose a two-stage adaptive robust optimisation approach for batch scheduling problems under uncertainties such as processing time and order demand.

Few studies focus on developing schedules considering batching and lot streaming simultaneously, although they are well studied in isolation. The main goal of this study is to fill this gap.

3. Model

3.1. Problem description

Consider a 2-HFS: m_1 and m_2 ($m_2 \geq m_1$) identical parallel, capacitated machines are in stage 1 and 2, respectively. A set of customer orders, consisting of products, of varying weights (priorities) is released at the beginning of the time horizon. Each product can be divided into several variable size sublots. Multiple sublots, possibly from different products, can be processed simultaneously on the same machine as a batch if they belong to the same product family and their total size does not exceed the machine capacity. A sequence-dependent setup is required for changeovers at each machine. We seek to determine the number and size of sublots for each product and the sequences for each sublot such that TWCT for product sublots is minimised. Using the notation of Graham et al. (1979), this problem can be denoted by $HF2|$ p-batch, incompatible, split, $s_{ij}| \sum TWCT$. A special case of the problem, $HF2|| \sum TWCT$, is proven to be NP-hard (Tang, Xuan, and Liu 2006). Hence, the more general problem in this paper is also NP-hard.

3.2. Formulation

We formulate the integrated batching and lot streaming problem in a 2-HFS as an MILP. The notation used in the mathematical model is defined as follows:

Sets

P	Set of products; $p = 1, 2, \ldots |P|$
S	Set of flow shop stages; $s = 1,2$
W_s	Set of machines in stage s; indexed by $k, m = 1, 2, \ldots |W_s|$
B	Set of batch positions; $j, b = 0, 1, 2, \ldots |B|$. Subset $B_m \subseteq B$ denotes the batch positions on machine $m \in W_s$
N	Set of sublots; $\alpha = 1, 2, \ldots |N|$
F	Set of product families; $f, g = 0, 1, 2, \ldots |F|$

A maximum number of sublots $|N|$ is given to any product p by a decision maker. $|B|$ is the maximum number of batches that any machine can process. Batch position 0 is a dummy batch position to which only dummy product family 0 can be assigned.

Parameters

K_s	Capacity of each identical machine in stage s
D_p	Product p demand
M_1, M_2, M_3	Large positive numbers
t_{fs}	Processing time of family f in stage s
w_p	Product p weight
ρ_{pf}	$= 1$ if product p belongs to family f, 0 otherwise
τ_{fg}	Setup time between families f and g

Variables

$n_{p\alpha smb}$	Size of sublot α of product p in batch position b on machine m in stage s
A_{fsmb}	Starting time of batch position b (in family f) on machine m in stage s
δ_{fsmb}	Completion time of batch position b (in family f) on machine m in stage s
$C_{p\alpha smb}$	Completion time of sublot α of product p assigned to batch position b on machine m in stage s
$u_{p\alpha}$	1 if sublot α of product p is produced, 0 otherwise
$x_{p\alpha smb}$	1 if sublot α of product p is assigned to batch position b on machine m in stage s, 0 otherwise
y_{fsmb}	1 if batch position b (processing family f) on machine m in stage s is used, 0 otherwise
$z_{p\alpha kjmb}$	1 if sublot α of product p is assigned to batch position j on machine k in stage 1 and batch position b on machine m in stage 2, 0 otherwise
$\beta^{sm}_{fbg,b+1}$	1 if family g in batch position $(b+1)$ is processed immediately after family f in batch position b on machine m stage s, 0 otherwise

The objective function, to minimise TWCT for product sublots, is given by:

$$\min TWCT = \sum_{p\in P}\sum_{\alpha\in N}\sum_{m\in W_2}\sum_{b\in B_m} w_p C_{p\alpha 2mb}. \tag{1}$$

The schedule is anchored by a dummy batch position before the first batch on each machine, filled by the dummy family 0:

$$y_{0sm0} = 1 \quad \forall s \in S,\ m \in W_s. \tag{2}$$

Batch positions are assigned consecutively with no empty batches interspersed:

$$y_{0sm0} - \sum_{g\in F} y_{gsm1} \geq 0 \quad \forall s \in S,\ m \in W_s, \tag{3}$$

$$\sum_{f\in F} y_{fsmb} - \sum_{g\in F} y_{gsm,b+1} \geq 0 \quad \forall s \in S,\ m \in W_s,\ b \in B \setminus \{|B|\}. \tag{4}$$

Any product sublot with family f cannot be assigned to a batch position if the family is not assigned to the same batch position.

$$\rho_{pf} x_{p\alpha smb} - y_{fsmb} \leq 0 \quad \forall p \in P,\ \alpha \in N,\ f \in F,\ s \in S,\ m \in W_s,\ b \in B. \tag{5}$$

The assignment of family sequences between two consecutive batch positions is subject to:

$$y_{fsmb} + y_{gsm,b+1} - \beta^{sm}_{fbg,b+1} \leq 1 \quad \forall f \in F \cup \{0\},\ g \in F, s \in S,\ m \in W_s, b \in (B \cup \{0\}) \setminus \{|B|\}. \tag{6}$$

Every product sublot must be assigned to a position in stage 1:

$$\sum_{k \in W_1} \sum_{j \in B} x_{p\alpha 1kj} = u_{p\alpha} \quad \forall p \in P,\ \alpha \in N. \tag{7}$$

Every sublot produced in stage 1 must be assigned to a position in stage 2:

$$\sum_{m \in W_2} \sum_{b \in B} x_{p\alpha 2mb} \geq \sum_{k \in W_1} \sum_{j \in B} x_{p\alpha 1kj} - M_2(1 - u_{p\alpha}) \quad \forall p \in P,\ \alpha \in N, \tag{8}$$

$$\sum_{m \in W_2} \sum_{b \in B} x_{p\alpha 2mb} \leq \sum_{k \in W_1} \sum_{j \in B} x_{p\alpha 1kj} + M_2 u_{p\alpha} \quad \forall p \in P,\ \alpha \in N. \tag{9}$$

Batch positions across two stages for a sublot are connected by:

$$x_{p\alpha 1kj} + x_{p\alpha 2mb} - z_{p\alpha kjmb} \leq 1 \quad \forall p \in P,\ \alpha \in N,\ k \in W_1,\ m \in W_2. \tag{10}$$

The production size of unused batch positions is equal to 0:

$$n_{p\alpha smb} \leq M_1 x_{p\alpha smb} \quad \forall p \in P,\ \alpha \in N,\ s \in S,\ m \in W_s,\ b \in B. \tag{11}$$

Batched quantities are bounded by 0 and the machine's capacity K_s:

$$\sum_{p \in P} \sum_{\alpha \in N} n_{p\alpha smb} \geq 0.001 \sum_{f \in F} y_{fsmb} \quad \forall s \in S,\ m \in W_s,\ b \in B, \tag{12}$$

$$\sum_{p \in P} \sum_{\alpha \in N} n_{p\alpha smb} \leq K_s \sum_{f \in F} y_{fsmb} \quad \forall s \in S,\ m \in W_s,\ b \in B. \tag{13}$$

All customer demand must be assigned to stage 1, ensuring that all demand is satisfied:

$$\sum_{\alpha \in N} \sum_{m \in W_1} \sum_{b \in B} n_{p\alpha 1mb} = D_p \quad \forall p \in P. \tag{14}$$

The size of a sublot of product p that is processed in stage 1 and stage 2 should be consistent—i.e. the size of a sublot of product p processed in stage 2 should equal the size of a sublot of product p processed in stage 1. However, two sublots of a product can be processed in one batch position so that the actual sublot size of product p on machine m in stage s is determined by $\sum_{\alpha \in N} n_{p\alpha smb}$. Products produced in stage 2 satisfy customer demand:

$$n_{p\alpha 1kj} \geq \sum_{m \in W_2} \sum_{b \in B} n_{p\alpha 2mb} + M_1 \left(\sum_{m \in W_2} \sum_{b \in B} z_{p\alpha kjmb} - \sum_{m \in W_2} \sum_{b \in B} x_{p\alpha 2mb} \right) \quad \forall p \in P,\ \alpha \in N,\ k \in W_1,\ j \in B, \tag{15}$$

$$n_{p\alpha 1kj} \leq \sum_{m \in W_2} \sum_{b \in B} n_{p\alpha 2mb} \quad \forall p \in P,\ \alpha \in N,\ k \in W_1,\ j \in B, \tag{16}$$

$$n_{p\alpha 1kj} \leq M_1 \sum_{m \in W_2} \sum_{b \in B} z_{p\alpha kjmb} \quad \forall p \in P,\ \alpha \in N,\ k \in W_1,\ j \in B. \tag{17}$$

The completion time of product sublot α processed by batch position b on machine m in stage s equals the completion time of the batch position that is used for family f:

$$C_{p\alpha smb} \geq \delta_{fsmb} + M_3(x_{p\alpha smb} - 1) \quad \forall p \in P,\ \alpha \in N,\ s \in S,\ f \in F,\ m \in W_s,\ b \in B. \tag{18}$$

The completion time of a batch position on any machine in stage s is its starting time plus the associated product family's processing time in stage s:

$$\delta_{fsmb} \geq A_{fsmb} + t_{fs} + M_3(y_{fsmb} - 1) \quad \forall f \in F \cup \{0\},\ s \in S,\ m \in W_s,\ b \in B \cup \{0\}. \tag{19}$$

The setup for a sublot on a machine is subject to:

$$A_{gsm,b+1} \geq \delta_{fsmb} + \beta^{sm}_{fbg,b+1}\tau_{fg} - M_3(1 - \beta^{sm}_{fbg,b+1})$$
$$\forall f \in F \cup \{0\},\ g \in F,\ s \in S,\ m \in W_s,\ b \in (B \cup \{0\}) \setminus \{|B|\}, \tag{20}$$
$$A_{g2m,b+1} \geq C_{p\alpha 1kj} + \beta^{sm}_{fbg,b+1}\tau_{fg} - M_3(2 - \beta^{sm}_{fbg,b+1} - z_{p\alpha kjmb})$$
$$\forall p \in P,\ \alpha \in N,\ k \in W_1,\ m \in W_2,\ f \in F \cup \{0\},\ g \in F,\ b \in (B \cup \{0\}) \setminus \{|B|\}. \tag{21}$$

Constraint (20) ensures that the overlapping of sublots on the same machine is prevented. Sublots processed in batch position $b+1$ on machine m in stage s can start only after sublots assigned to batch position b on machine m in stage s have been completed. Constraint (21) requires that sublots can only start in stage 2 after their completion in stage 1. Variable type constraints are given by

$$x_{p\alpha smb},\ y_{fsmb},\ z_{pkjmb},\ u_{p\alpha},\ \beta^{sm}_{fbg,b+1} \in \{0, 1\}$$
$$\forall p \in P,\ \alpha \in N,\ f, g \in F \cup \{0\},\ s \in S,\ k,\ m \in W_s,\ j, b \in B \cup \{0\}, \tag{22}$$

$$n_{p\alpha smb},\ \delta_{fsmb},\ A_{fsmb},\ C_{p\alpha smb} \geq 0,$$
$$\forall p \in P,\ \alpha \in N,\ f \in F \cup \{0\},\ s \in S,\ m \in W_s,\ b \in B. \tag{23}$$

3.3. Valid inequalities

To improve tractability, two symmetry-breaking valid inequalities are added:

$$y_{f211} \geq y_{f111} \quad \forall f \in F \cup \{0\}, \tag{24}$$
$$\sum_{\alpha \in N} x_{p\alpha 111} \geq \sum_{\alpha \in N} x_{p\alpha 211} \quad \forall p \in P. \tag{25}$$

If a product family is assigned to the first batch position on the first machine in stage 1, it must be assigned to a batch position on machine m in stage 2. Without loss of generality, we assign this product family to the first batch position on the first machine in stage 2 in (24). Similarly, (25) specifies that product p sublots in the first batch position on the first machine in stage 2 are from the sublots in the first batch position on the first machine in stage 1.

3.4. Lower bound

To evaluate solution quality, we develop a lower bound. We first aggregate product demands $\sum_p D_p\rho_{pf}$ for each product family *f*. Then these aggregated products are split based on stage 2's capacity to form batches. We assume stage 1 has infinite number of machines and all products are processed at the beginning of the time horizon. Hence, the start time of each batch in stage 2 equals the processing time t_{f1} of the product family of this batch. If there is more than one product in a batch, then we assign the weights of all products in this batch as $min\{w_p\}$. Otherwise, product weight in this batch is w_p. Next, we pick batches according to increasing t_{f2}/w_p and assign them to available machines in stage 2, ignoring setup times.

4. Heuristics

The heuristics must handle sublot formation and scheduling. Sublot formation involves product sequencing and splitting, while sublot scheduling consists of sublot sequencing and assigning formed sublots on machines. To sequence products, we adapt three common approaches found in scheduling literature:

- *Random Key Method* (*RK*): Sample random numbers from U [0, 1000] that are mapped to product numbers; the numbers are sorted according to these keys to create a sequence. Assume three products and the random string [42, 38, 34] is sampled for the products indexed [1, 2, 3]. Sorting the random numbers and the product indexes together results in the sequence [3, 2, 1].
- *Weighted Shortest Processing Time* (*WSPT*): The product with the smallest weighted processing time in stage 1 is selected (Li et al. 2015). The processing time for product p depends on its family type. Therefore, the weighted

processing time for product p in stage 1, denoted wt_p, is defined as:

$$wt_p = \frac{t_{f_p 1}}{w_p} \quad \forall p \in P, \tag{26}$$

where f_p represents the family type of product p and w_p is product p's weight.

- *Johnson's Rule* (*JR*): Select the unscheduled product with the shortest processing time in either stage, denoted $t_{f_p s}$. If the shortest time is in stage 1, schedule the product as early as possible in the sequence. Otherwise, schedule the product as late as possible. Remove the product from the list of unscheduled products. Repeat until all products have been scheduled. If there is a tie for shortest processing time, arbitrarily choose one of the products.

The processes of splitting sequenced products to form sublots and assigning and sequencing the formed sublots on machines are completed simultaneously. We begin with splitting the first product according to our splitting rules. Three methods are used for splitting products:

- *Stage 1's Capacity* ($C1$): If product p's demand is greater than stage 1's capacity, split this product using stage 1's capacity. Otherwise, sublot size equals product p's demand.
- *Stage 2's Capacity* ($C2$): If product p's demand is greater than stage 2's capacity, split this product using stage 2's capacity. Otherwise, sublot size equals product p's demand.
- *Random Sublot Size* (R): If product p's demand is greater than stage 1's capacity, split it using a random number generated using uniform distribution $[0, K_1]$. Otherwise, sublot size equals equals product p's demand.

Sublots for the first product are assigned to appropriate batch positions on available machines in stage 1 to minimise the completion time of sublots. The rule of selecting sublots is picking in a decreasing order of sublot sizes, breaking ties arbitrarily.

For the next product p to be split, check if there is a partial batch that is not full and of the same family. If so, fill out this batch position with min{left capacity of this batch position, D_p}. Then, split this product into sublots and assign formed sublots on machines according to the procedures described above for the first product.

As stage 2's capacity is less than stage 1's, some formed batches in stage 1 may have to be split for stage 2. The split batches, now sublots, are assigned to available machines in stage 2 to minimise TWCT. Different heuristics are tested by applying the combinations of proposed product-sequencing methods and splitting rules (see Table 1).

After generating the schedule for all products in each stage, we will conduct random local search (RLS) for both stage 1 and stage 2 individually to generate more possible schedules. New schedules are generated by swapping two randomly selected batches on machines in stage 1 and stage 2 individually. Figure 3 shows the procedure of randomly swapping two batch positions in stage 1. We randomly select two batch positions: [P1(3.4), P3(3.8)] and [P3(7.2)]. In these two batches, one has two sublots (3.4 tons of product 1 and 3.8 tons of product 3) and the other has one sublot (7.2 tons of product 3). Then, we swap the sublots in these two batch positions to form a new schedule for stage 1.

5. Computational experiments

To investigate the performance of the proposed MILP and heuristic algorithms, 40 instances are generated (Table 2). The first 10 instances are smaller than the last 30 instances in terms of product demands and the number of machines in stage 2. In Instances 1-10 (11-40), all instances have one machine in stage 1 with a capacity of 7.2 units and two (three) identical parallel vessels each with capacity of 4 in stage 2. Products in the same order have the same weights: random integers between 1 and 3 (i.e. $w_p \sim DU$ [1, 3]). Table 3 provides the number of orders and the number of products in each order for all instances. As shown in Table 3, 2-3 products are studied in each order for painting cargo containers (CCs), ship hulls (SHs), or industrial structures (ISs) with probability 0.05, 0.25, and 0.7, respectively. In Instances 1-10, demand for primer for all orders are randomly generated according to uniform distributions U [1, 15]. For Instances 11-40, primer

Table 1. Heuristic description.

Sequencing			
Splitting	*RK*	*WSPT*	*JR*
$C1$	$C1R$	$C1W$	$C1J$
$C2$	$C2R$	$C2W$	$C2J$
R	RR	RW	RJ

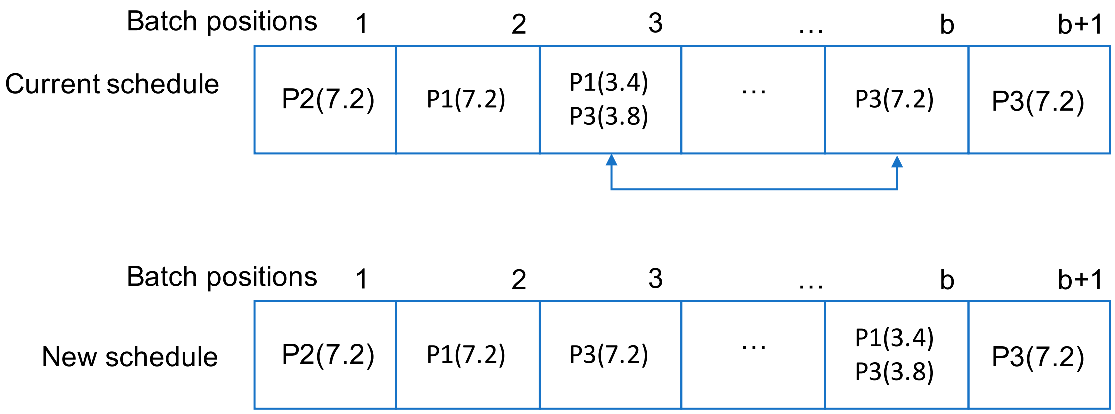


Figure 3. An example of randomly swapping two batch positions in stage 1.

Table 2. Experimental design for numerical instances.

Parameter	Value description
Order type	CCs with probability of 0.05 SHs with probability of 0.25 ISs with probability of 0.7
Weight	*DU* [1,3]
Primer demand (Instance 1- 10)	*U* [1,15]
Primer Demand (Instance 11- 30)	CCs: *U* [12,25] SHs: *U* [12,45] ISs: *U* [7,35]

Table 3. Processing times and setup times for numerical instances.

Instances	Number of orders	Number of products in each order
1—5	1	2
6–10	1	3
11–15	1	2
16–20	1	3
21–25	2	(Order1, 2), (Order 2, 2)
26–30	2	(Order1, 2), (Order 2, 3)
31–35	2	(Order1, 3), (Order 2, 2)
36–40	2	(Order1, 3), (Order 2, 3)

Table 4. Processing times and setup times for numerical instances.

	Processing Time		Setup Time		
Family	Stage 1	Stage 2	Family 1	Family 2	Family 3
1	2.14	3.36	0	0.5	0.7
2	2.73	1.52	0.3	0	1
3	1.67	2.44	0.8	0.8	0

demand in CC, SH and IS orders are generated using real demand distributions which are *U* [12, 25], *U* [12, 45] and *U* [7, 35], respectively, while demand for top coat and undercoat paint are 50% of and 20% of the ordered primer quantity, respectively. Primer, top coat paint, and undercoat paint belong to three incompatible product families. The processing times and setup times (Table 4) are common for all instances.

The optimisation model and algorithms were implemented using JuMP and Gurobi 7.0.1 on Clemson University's high-performance computing resource, the Palmetto Cluster, which has Intel® Xenon® CPU, 16 core processors @ 2.65 GHz and 128 GB RAM.

Table 5. Gurobi results (1hr) for MILP experiments and LBs.

	Original MILP			$\text{MILP}_{2\text{VI}}$			LBs	
Instance	$TWCT_O$	LB_O	Gap_O	$TWCT_{2VI}$	LB_{2VI}	Gap_{2VI}	LB_H	Δ
1	18.52	18.52	0.0%	18.52	18.52	0.0%	9.16	50.5%
2	48.98	48.98	0.0%	48.98	48.98	0.0%	16.18	67.0%
3	37.51	37.51	0.0%	37.51	37.51	0.0%	16.18	56.9%
4	48.98	48.98	0.0%	48.98	48.98	0.0%	16.18	67.0%
5	48.98	48.98	0.0%	48.98	48.98	0.0%	16.18	67.0%
6	63.59	59.09	7.1%	63.59	63.59	0.0%	23.20	63.5%
7	29.70	29.70	0.0%	29.70	29.70	0.0%	15.26	48.6%
8	63.59	59.88	5.8%	63.59	63.59	0.0%	23.20	63.5%
9	51.13	33.09	35.3%	51.13	51.13	0.0%	23.20	54.6%
10	29.70	29.70	0.0%	29.70	29.70	0.0%	15.26	48.6%
11	195.69	10.08	94.8%	196.32	13.45	93.1%	98.88	49.5%
12	444.72	26.88	94.0%	652.62	3.04	99.5%	281.96	36.6%
13	209.16	16.80	92.0%	201.40	16.80	91.7%	123.88	38.5%
14	269.16	26.88	90.0%	265.32	26.88	89.9%	170.28	35.8%
15	170.70	20.16	88.2%	174.87	4.56	97.4%	107.16	37.2%
16	120.10	0.00	100.0%	99.30	0.00	100.0%	67.76	31.8%
17	209.43	0.00	100.0%	161.48	0.00	100.0%	74.16	54.1%
18	121.00	0.00	100.0%	127.46	0.00	100.0%	74.16	38.7%
19	190.16	0.00	100.0%	201.74	0.00	100.0%	122.68	35.5%
20	239.07	0.00	100.0%	190.13	0.00	100.0%	111.38	41.4%
21	595.59	0.00	100.0%	545.78	0.00	100.0%	271.64	50.2%
22	747.05	0.00	100.0%	731.50	0.00	100.0%	352.04	51.9%
23	1164.73	0.00	100.0%	1013.79	0.00	100.0%	436.40	57.0%
24	367.71	0.00	100.0%	305.13	0.00	100.0%	128.78	57.8%
25	1218.18	0.00	100.0%	1209.82	0.00	100.0%	523.82	56.7%
26	220.35	0.00	100.0%	199.76	0.00	100.0%	96.12	51.9%
27	978.54	0.00	100.0%	1053.34	0.00	100.0%	483.24	50.6%
28	1469.84	0.00	100.0%	1414.64	0.00	100.0%	607.68	57.0%
29	928.25	0.00	100.0%	999.86	0.00	100.0%	417.66	55.0%
30	624.94	0.00	100.0%	665.49	0.00	100.0%	245.32	60.7%
31	285.10	0.00	100.0%	425.8	0.00	100.0%	126.94	55.5%
32	–	–	–	–	–	–	802.04	–
33	755.39	0.00	100.0%	895.05	0.00	100.0%	339.58	55.0%
34	1337.34	0.00	100.0%	1258.72	0.00	100.0%	530.24	57.9%
35	1616.12	0.00	100.0%	1397.6	0.00	100.0%	406.48	70.9%
36	991.89	0.00	100.0%	1111.39	0.00	100.0%	373.16	62.4%
37	3359.01	0.00	100.0%	1627.82	0.00	100.0%	620.78	61.9%
38	2272.91	0.00	100.0%	2474.04	0.00	100.0%	791.28	65.2%
39	1574.21	0.00	100.0%	1109.37	0.00	100.0%	451.46	59.3%
40	565.58	0.00	100.0%	504.46	0.00	100.0%	220.56	56.3%

5.1. MILP results and quality of the LB

Per company requirements, each MILP instance can run for at most 1 hour, given the need to make twice-a-day production schedules. In our experiments, optimal solutions were only found for seven small instances (1-5, 7 and 10) using the original MILP formulation. All other instances stopped at the 1-hour limit before finding an optimal solution. For Instance 32, no feasible solution was found. Table 5 summarises Gurobi solutions. The 2nd and 5*th* columns provide the current incumbents (i.e. the best objective values) found by Gurobi for the original MILP and the MILP with two valid inequalities ($\text{MILP}_{2\text{VI}}$). Instance 6, 8 and 9 found optimal solutions using the valid inequalities. For Instances 11-40, although the use of the valid inequalities did not improve solutions for all instances, the solutions were improved 2.1%, on average. Hence, further experiments will be conducted using the valid inequalities unless otherwise noted.

The optimality gap, the difference between the current incumbent solution and Gurobi's LB, is provided in the 3rd and 6th columns in Table 5. A 0%(100%) optimality gap of 0 means Gurobi found the optimal solution (no LB). Table 5 shows that the LBs returned by Gurobi for Instances 11-40 are poor for both MILP and $\text{MILP}_{2\text{VI}}$. In instances with more than two products (16-40), the optimality gap is always 100%. The average gaps of all instances using MILP and $\text{MILP}_{2\text{VI}}$ are 75.4% and 73.6%, respectively.

Let Δ denote the gap between the minimum objective value obtained from the original MILP and $\text{MILP}_{2\text{VI}}$ (ie., $TWCT_O$ and $TWCT_{2VI}$) and our calculated LB (LB_H) such that $\Delta = (\min\{TWCT_O, TWCT_{2VI}\} - LB_H)/\min\{TWCT_O, TWCT_{2VI}\}$. The

Table 6. Min, Mean, Max objective value, average Gap_H, and computational time of heuristics.

	C1R					*C2R*					*RR*				
Instances	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)
1	18.52	18.52	18.52	50.5%	2.17	22.67	22.67	22.67	59.6%	2.58	18.52	18.52	18.52	50.5%	2.40
2	48.98	48.98	48.98	67.0%	3.45	64.69	64.69	64.69	75.0%	3.77	48.98	48.98	48.98	67.0%	3.86
3	37.51	37.51	37.51	56.9%	3.39	48.99	48.99	48.99	67.0%	3.59	37.51	37.51	37.51	56.9%	3.12
4	48.98	48.98	48.98	67.0%	3.45	64.69	64.69	64.69	75.0%	3.68	48.98	48.98	48.98	67.0%	3.86
5	48.98	48.98	48.98	67.0%	3.48	64.69	64.69	64.69	75.0%	3.74	63.81	63.81	63.81	74.6%	4.23
6	63.59	63.59	63.59	63.5%	4.30	84.90	84.90	84.90	72.7%	4.69	63.59	63.59	63.59	63.5%	4.40
7	29.70	29.70	29.70	48.6%	2.61	34.99	34.99	34.99	56.4%	2.74	29.70	29.70	29.70	48.6%	2.36
8	63.59	63.59	63.59	63.5%	4.39	65.75	65.75	65.75	64.7%	3.84	63.59	63.59	63.59	63.5%	4.10
9	51.13	51.13	51.13	54.6%	3.59	65.75	65.75	65.75	64.7%	3.79	51.13	51.13	51.13	54.6%	3.89
10	29.70	29.70	29.70	48.6%	2.63	34.99	34.99	34.99	56.4%	2.71	29.70	29.70	29.70	48.6%	2.63
11	176.39	176.39	176.39	43.9%	7.04	208.28	208.57	208.78	52.6%	7.05	177.67	193.02	214.59	48.8%	10.13
12	438.34	438.34	438.34	35.7%	8.12	619.70	620.94	621.70	54.6%	8.56	442.40	498.48	535.10	43.4%	12.48
13	193.68	193.68	193.68	36.0%	7.25	274.52	274.98	275.39	54.9%	7.24	193.68	212.31	227.81	41.7%	10.46
14	267.54	268.21	268.54	36.5%	5.75	358.48	358.86	359.48	52.5%	6.36	268.54	268.54	268.54	36.6%	8.04
15	223.47	223.49	223.56	52.1%	4.11	192.57	192.57	192.57	44.4%	2.48	223.47	223.61	224.10	52.1%	4.75
16	99.30	99.30	99.30	31.8%	3.14	129.90	129.90	129.90	47.8%	3.41	99.30	99.30	99.30	31.8%	3.62
17	124.46	124.60	124.61	40.5%	5.79	154.22	154.22	154.22	51.9%	6.50	124.61	125.51	127.96	40.9%	6.97
18	104.58	104.58	104.58	29.1%	5.24	154.22	154.22	154.22	51.9%	6.24	104.58	107.19	115.43	30.8%	6.69
19	185.42	185.67	186.02	33.9%	4.65	254.92	256.15	256.92	52.1%	4.73	185.42	186.24	187.02	34.1%	5.60
20	154.90	154.90	154.90	28.1%	6.78	242.89	242.89	242.89	54.1%	7.76	154.90	175.98	188.53	36.7%	9.06
21	506.87	507.24	507.79	46.4%	10.35	535.61	542.41	550.89	49.9%	9.62	516.43	564.02	610.17	51.8%	12.99
22	599.37	606.64	610.60	42.0%	14.99	782.31	800.67	816.38	56.0%	15.50	759.27	827.23	890.35	57.4%	20.56
23	742.74	742.74	742.74	41.2%	10.75	1043.3	1063.17	1077.83	59.0%	10.86	857.50	969.03	1046.37	55.0%	15.46
24	245.82	246.30	246.44	47.7%	7.37	271.75	275.06	279.41	53.2%	5.33	247.88	254.72	269.24	49.4%	9.23
25	878.85	882.60	885.51	40.7%	12.65	1121.74	1140.66	1159.00	54.1%	12.15	1068.18	1128.52	1184.95	53.6%	15.31
26	174.21	174.21	174.21	44.8%	7.97	185.38	185.50	186.30	48.2%	6.49	174.21	174.81	176.93	45.0%	7.95
27	799.50	806.32	810.90	40.1%	11.80	1048.16	1072.79	1099.84	55.0%	8.08	866.94	968.50	1048.24	50.1%	14.87
28	1145.54	1154.20	1162.02	47.4%	15.59	1356.48	1370.50	1392.62	55.7%	14.11	1270.82	1375.86	1504.82	55.8%	18.31
29	652.87	659.02	662.12	36.6%	9.56	878.75	889.22	902.95	53.0%	9.15	717.72	737.96	771.24	43.4%	11.00
30	444.92	447.67	451.52	45.2%	10.28	640.37	652.37	661.63	62.4%	7.71	485.42	524.22	559.91	53.2%	12.77
31	204.05	204.49	206.60	37.9%	6.27	251.54	258.51	262.83	50.9%	7.28	204.05	208.94	217.43	39.2%	7.70
32	1355.95	1366.96	1371.08	41.3%	16.77	1849.43	1880.71	1909.23	57.4%	13.88	1696.63	1841.27	1970.89	56.4%	19.93
33	578.28	580.77	584.52	41.5%	9.70	697.11	706.66	716.55	51.9%	10.03	578.88	606.96	628.46	44.1%	11.77
34	924.62	928.19	934.64	42.9%	12.41	1140.30	1154.84	1176.62	54.1%	8.62	1029.42	1130.26	1203.38	53.1%	17.50
35	685.99	689.47	691.49	41.0%	8.21	963.68	978.00	986.68	58.4%	8.71	784.14	869.08	923.52	53.2%	16.91
36	789.58	796.60	800.00	53.2%	11.96	828.24	853.12	868.47	56.3%	6.92	838.55	882.54	924.30	57.7%	13.20
37	1083.06	1085.16	1095.63	42.8%	14.53	1300.86	1321.69	1329.58	53.0%	13.71	1307.33	1390.31	1470.94	55.3%	16.95
38	1355.74	1368.35	1379.48	42.2%	17.65	1860.64	1887.61	1902.64	58.1%	10.34	1659.95	1826.03	1908.28	56.7%	20.25
39	788.76	790.01	794.24	42.9%	17.71	1023.08	1038.22	1043.78	56.5%	16.20	970.13	1039.97	1103.90	56.6%	19.68
40	381.51	383.54	384.87	42.5%	12.15	451.16	462.10	471.60	52.3%	7.78	462.91	486.99	521.08	54.7%	14.64

	C1W					*C2W*					*RW*				
Instances	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)
1	18.52	18.52	18.52	50.5%	2.32	22.67	22.67	22.67	59.6%	2.67	18.52	18.52	18.52	50.5%	1.37
2	48.98	48.98	48.98	67.0%	3.38	64.69	64.69	64.69	75.0%	3.40	48.98	48.98	48.98	67.0%	2.28
3	37.51	37.51	37.51	56.9%	3.16	48.99	48.99	48.99	67.0%	3.22	37.51	37.51	37.51	56.9%	1.95
4	48.98	48.98	48.98	67.0%	3.31	64.69	64.69	64.69	75.0%	3.38	48.98	48.98	48.98	67.0%	2.32
5	48.98	48.98	48.98	67.0%	3.29	64.69	64.69	64.69	75.0%	3.75	63.81	63.81	63.81	74.6%	2.61
6	63.59	63.59	63.59	63.5%	4.26	84.90	84.90	84.90	72.7%	4.82	63.59	63.59	63.59	63.5%	2.75
7	29.70	29.70	29.70	48.6%	2.63	34.99	34.99	34.99	56.4%	2.93	29.70	29.70	29.70	48.6%	1.58
8	63.59	63.59	63.59	63.5%	4.13	65.75	65.75	65.75	64.7%	4.00	63.59	63.59	63.59	63.5%	2.90
9	51.13	51.13	51.13	54.6%	3.64	65.75	65.75	65.75	64.7%	3.91	51.13	51.13	51.13	54.6%	2.79
10	29.70	29.70	29.70	48.6%	2.97	34.99	34.99	34.99	56.4%	2.96	29.70	29.70	29.70	48.6%	2.90

(*Continued*).

Table 6. Continued.

	C1W					*C2W*					*RW*				
Instances	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)
11	176.39	176.39	176.39	43.9%	7.81	208.28	208.49	208.78	52.6%	7.11	177.67	191.30	207.46	48.3%	11.63
12	438.34	438.34	438.34	35.7%	9.37	619.70	620.75	621.70	54.6%	8.49	449.76	494.52	528.76	43.0%	13.14
13	193.68	193.68	193.68	36.0%	8.46	274.39	274.84	275.02	54.9%	7.76	193.68	210.21	232.52	41.1%	12.16
14	267.54	268.27	268.54	36.5%	6.69	358.48	358.60	359.48	52.5%	5.75	268.54	268.54	268.54	36.6%	9.39
15	223.47	223.48	223.56	52.1%	4.75	192.57	192.57	192.57	44.4%	3.88	223.47	223.56	223.95	52.1%	6.12
16	99.30	99.30	99.30	31.8%	3.92	129.90	129.90	129.90	47.8%	4.03	99.30	99.30	99.30	31.8%	4.85
17	124.61	124.61	124.61	40.5%	6.51	154.22	154.22	154.22	51.9%	6.52	124.61	126.39	129.34	41.3%	8.66
18	104.58	104.58	104.58	29.1%	5.69	154.22	154.22	154.22	51.9%	6.29	104.58	107.42	114.09	31.0%	8.32
19	185.42	186.17	186.26	34.1%	5.42	255.32	256.00	256.88	52.1%	5.61	185.68	186.30	187.56	34.1%	7.04
20	154.90	154.90	154.90	28.1%	6.72	242.89	242.89	242.89	54.1%	8.05	154.90	178.19	192.72	37.5%	10.96
21	506.87	506.87	506.87	46.4%	12.25	534.73	538.73	541.07	49.6%	9.99	507.79	529.53	544.70	48.7%	15.44
22	599.37	604.72	608.12	41.8%	15.54	782.31	794.03	799.76	55.7%	15.64	730.87	797.80	852.42	55.9%	22.17
23	742.74	742.74	742.74	41.2%	11.63	1055.02	1063.77	1074.11	59.0%	11.20	837.72	897.69	965.50	51.4%	17.24
24	245.82	245.84	246.41	47.6%	8.63	271.75	273.54	274.49	52.9%	7.98	245.82	248.70	250.00	48.2%	7.63
25	878.85	879.55	882.45	40.4%	12.77	1104.78	1125.27	1141.34	53.4%	12.21	955.51	1056.79	1142.91	50.4%	12.09
26	174.21	174.21	174.21	44.8%	7.72	185.38	185.99	186.30	48.3%	7.81	174.21	176.43	179.71	45.5%	6.89
27	799.50	804.84	811.16	40.0%	13.38	1035.96	1059.94	1084.64	54.4%	12.10	892.58	989.60	1041.44	51.2%	11.58
28	1141.76	1159.37	1169.02	47.6%	14.29	1340.36	1361.14	1391.82	55.4%	14.24	1300.28	1395.68	1505.56	56.5%	14.36
29	657.84	658.15	658.26	36.5%	9.16	874.50	895.69	908.30	53.4%	10.75	708.53	717.92	731.75	41.8%	8.64
30	445.54	446.28	447.25	45.0%	11.89	628.93	638.99	647.79	61.6%	11.97	453.38	487.79	518.60	49.7%	10.19
31	203.12	204.01	204.05	37.8%	7.52	258.42	261.50	264.11	51.5%	7.43	204.05	207.18	212.19	38.7%	7.31
32	1361.75	1372.04	1394.37	41.5%	17.18	1837.83	1852.13	1865.63	56.7%	16.38	1619.94	1738.17	1818.15	53.9%	20.42
33	578.88	580.33	583.16	41.5%	9.59	701.02	707.26	713.63	52.0%	10.17	584.16	597.33	606.42	43.2%	11.48
34	934.54	941.08	950.18	43.7%	13.54	1130.06	1146.31	1155.90	53.7%	12.91	1046.70	1130.92	1195.32	53.1%	17.25
35	685.99	685.99	685.99	40.7%	14.00	951.76	958.84	963.68	57.6%	13.26	732.34	789.02	835.70	48.5%	16.13
36	792.41	795.49	798.62	53.1%	11.08	853.84	863.08	872.13	56.8%	11.61	803.08	847.93	875.60	56.0%	13.79
37	1079.70	1081.81	1085.59	42.6%	14.10	1301.50	1321.55	1333.02	53.0%	14.07	1218.68	1329.13	1426.69	53.3%	13.82
38	1355.74	1360.11	1362.46	41.8%	16.34	1850.86	1868.72	1883.80	57.7%	15.58	1490.88	1704.66	1808.89	53.6%	15.09
39	786.96	788.82	789.86	42.8%	17.79	1012.95	1024.80	1032.08	55.9%	16.76	926.75	971.29	1034.89	53.5%	15.97
40	382.31	385.77	388.22	42.8%	12.51	450.46	452.73	454.95	51.3%	12.45	461.20	472.17	495.37	53.3%	12.54

	C1J					*C2J*					*RJ*				
Instances	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)
1	18.52	18.52	18.52	50.5%	1.34	22.67	22.67	22.67	59.6%	2.39	18.52	18.52	18.52	50.5%	2.48
2	48.98	48.98	48.98	67.0%	3.59	64.69	64.69	64.69	75.0%	3.39	48.98	48.98	48.98	67.0%	4.40
3	37.51	37.51	37.51	56.9%	3.12	48.99	48.99	48.99	67.0%	3.20	37.51	37.51	37.51	56.9%	3.62
4	48.98	48.98	48.98	67.0%	3.32	64.69	64.69	64.69	75.0%	3.37	48.98	48.98	48.98	67.0%	4.33
5	48.98	48.98	48.98	67.0%	3.24	64.69	64.69	64.69	75.0%	3.70	63.81	63.81	63.81	74.6%	4.82
6	63.59	63.59	63.59	63.5%	4.22	84.90	84.90	84.90	72.7%	4.80	63.59	63.59	63.59	63.5%	5.08
7	29.70	29.70	29.70	48.6%	2.61	34.99	34.99	34.99	56.4%	2.56	29.70	29.70	29.70	48.6%	2.99
8	63.59	63.59	63.59	63.5%	4.20	65.75	65.75	65.75	64.7%	3.93	63.59	63.59	63.59	63.5%	4.92
9	51.13	51.13	51.13	54.6%	3.45	65.75	65.75	65.75	64.7%	3.77	51.13	51.13	51.13	54.6%	4.69
10	29.70	29.70	29.70	48.6%	2.95	34.99	34.99	34.99	56.4%	2.87	29.70	29.70	29.70	48.6%	2.86
11	176.39	176.39	176.39	43.9%	7.80	208.28	208.51	208.78	52.6%	6.76	178.72	190.97	204.04	48.2%	10.56
12	438.34	438.34	438.34	35.7%	6.18	619.96	620.80	621.70	54.6%	6.01	440.90	489.38	538.56	42.4%	11.00
13	193.18	193.66	193.68	36.0%	7.21	274.52	274.88	275.39	54.9%	5.30	193.68	212.94	236.18	41.8%	7.20
14	267.54	268.27	268.54	36.5%	5.89	358.48	358.52	359.48	52.5%	4.28	268.54	268.54	268.54	36.6%	5.61
15	223.47	223.48	223.56	52.1%	4.22	192.57	192.57	192.57	44.4%	2.79	223.47	223.53	223.95	52.1%	3.56
16	99.30	99.30	99.30	31.8%	3.56	129.90	129.90	129.90	47.8%	2.78	99.30	99.30	99.30	31.8%	2.76
17	124.61	124.61	124.61	40.5%	5.71	154.22	154.22	154.22	51.9%	4.31	124.61	126.36	129.02	41.3%	7.37
18	104.58	104.58	104.58	29.1%	5.53	154.22	154.22	154.22	51.9%	4.31	104.58	108.83	116.69	31.9%	6.97
19	185.96	186.20	186.26	34.1%	4.89	254.92	256.03	256.88	52.1%	3.81	185.42	186.34	187.02	34.2%	6.19
20	154.90	154.90	154.90	28.1%	6.94	242.89	242.89	242.89	54.1%	5.48	154.90	178.14	189.83	37.5%	10.04
21	515.49	534.83	546.38	49.2%	11.00	568.16	601.26	622.62	54.8%	10.31	554.43	615.96	668.69	55.9%	15.57
22	603.02	605.05	608.57	41.8%	14.88	776.66	793.49	808.32	55.6%	10.78	728.16	804.24	852.71	56.2%	23.08
23	746.52	753.55	765.72	42.1%	11.22	1033.91	1061.91	1080.77	58.9%	12.09	864.11	975.33	1071.95	55.3%	17.83
24	253.01	253.60	254.83	49.2%	7.86	278.58	285.47	290.19	54.9%	5.77	253.82	263.38	276.04	51.1%	10.88
25	883.43	892.47	898.27	41.3%	8.20	1128.47	1166.23	1188.29	55.1%	11.60	1039.73	1133.51	1212.49	53.8%	17.22

(*Continued*).

Table 6. Continued.

	C1J					C2J					RJ				
Instances	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)	Min	Mean	Max	Gap_H	Time (s)
26	174.21	174.21	174.21	44.8%	5.95	185.38	185.87	186.30	48.3%	7.81	174.21	175.87	179.01	45.3%	10.06
27	799.50	805.08	811.68	40.0%	9.15	1039.64	1053.88	1075.36	54.1%	13.27	910.14	1000.66	1055.60	51.7%	17.53
28	1144.22	1154.23	1165.98	47.4%	10.94	1340.36	1361.43	1366.04	55.4%	14.66	1231.16	1391.72	1468.46	56.3%	21.70
29	678.68	681.62	685.41	38.7%	7.34	878.75	900.96	912.74	53.6%	10.25	737.70	744.67	758.55	43.9%	12.37
30	462.26	466.39	471.42	47.4%	8.30	655.10	669.63	680.12	63.4%	11.78	494.56	526.15	567.79	53.4%	15.16
31	217.91	218.18	219.57	41.8%	5.44	262.67	265.58	269.07	52.2%	7.22	217.91	227.32	235.45	44.2%	9.08
32	1492.14	1503.22	1525.75	46.6%	11.84	1878.69	1917.59	1950.56	58.2%	15.75	1743.92	1911.52	2044.15	58.0%	23.60
33	603.56	604.46	605.14	43.8%	7.39	698.59	713.48	729.27	52.4%	10.18	586.04	626.30	656.60	45.8%	13.07
34	931.66	941.79	954.84	43.7%	9.86	1138.18	1157.48	1180.44	54.2%	14.13	1014.42	1143.00	1208.30	53.6%	19.91
35	741.70	762.31	772.71	46.7%	9.48	1026.66	1089.08	1114.61	62.7%	13.73	873.25	934.02	988.04	56.5%	11.79
36	812.05	821.75	836.15	54.6%	8.89	818.91	826.91	843.29	54.9%	11.62	876.17	903.80	930.31	58.7%	15.82
37	1121.94	1141.33	1151.24	45.6%	10.94	1347.35	1374.28	1399.88	54.8%	14.88	1298.77	1416.95	1510.08	56.2%	13.52
38	1373.12	1389.36	1400.40	43.0%	12.21	1846.89	1888.37	1902.28	58.1%	16.49	1610.55	1730.72	1827.03	54.3%	23.25
39	816.83	829.03	840.87	45.5%	12.06	1084.82	1124.61	1158.26	59.9%	16.74	1016.43	1085.57	1158.52	58.4%	21.66
40	381.51	385.30	388.80	42.8%	9.25	453.80	458.03	468.64	51.8%	12.51	464.37	477.96	502.21	53.9%	16.83

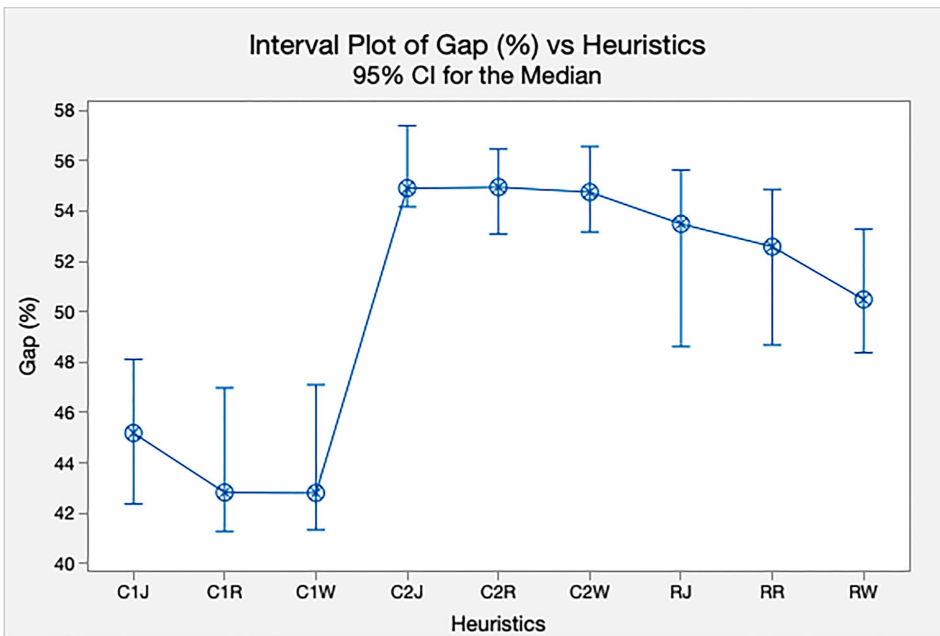
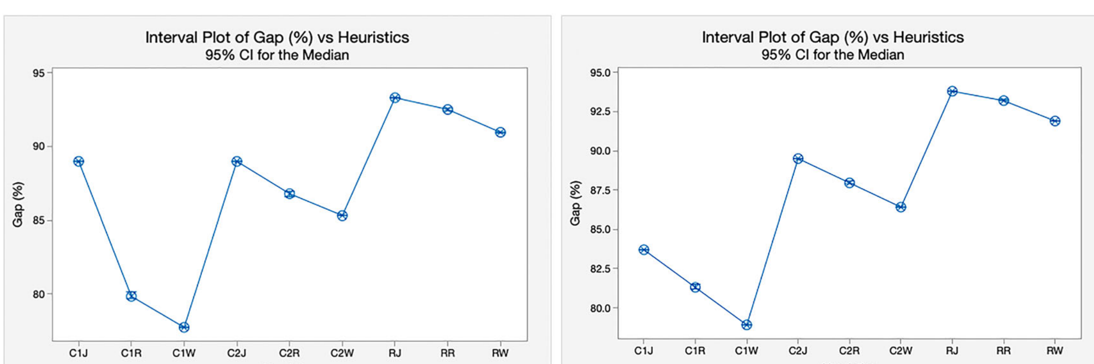


Figure 4. Confidence interval for the medians (40 instances).



(a) (b)

Figure 5. Confidence interval for the medians. (a) Instance O6 and (b) Instance O10.

Table 7. Input objective values and gurobi results.

Instance	V_{in}	V_{out}	$\frac{V_{in}-V_{out}}{V_{in}}$	LB_{GH}	Gap_{GH}	Instance	V_{in}	V_{out}	$\frac{V_{in}-V_{out}}{V_{in}}$	LB_{GH}	Gap_{GH}
11	176.39	170.04	3.6%	16.66	41.8%	26	174.21	154.90	11.1%	0.00	37.9%
12	438.34	405.58	7.5%	38.29	30.5%	27	809.48	787.54	2.7%	0.00	38.6%
13	193.68	189.65	2.1%	18.00	34.7%	28	1150.94	1103.82	4.1%	0.00	44.9%
14	267.54	258.70	3.3%	31.08	34.2%	29	657.84	613.28	6.8%	0.00	31.9%
15	223.47	161.97	27.5%	29.35	33.8%	30	446.02	423.15	5.1%	0.00	42.0%
16	99.30	99.30	0.0%	0.00	31.8%	31	204.05	194.39	4.7%	0.00	34.7%
17	124.61	124.44	0.1%	15.56	40.4%	32	1363.97	1312.11	3.8%	0.00	38.9%
18	104.58	103.84	0.7%	1.52	28.6%	33	578.88	548.09	5.3%	0.00	38.0%
19	185.96	185.96	0.0%	3.04	34.0%	34	929.44	890.70	4.2%	0.00	40.5%
20	154.90	154.90	0.0%	3.49	28.1%	35	685.99	660.35	3.7%	0.00	38.4%
21	506.87	434.61	14.3%	0.00	37.5%	36	793.50	756.44	4.7%	0.00	50.7%
22	601.51	569.36	5.3%	0.00	38.2%	37	1081.18	992.40	8.2%	0.00	37.4%
23	742.74	723.42	2.6%	0.00	39.7%	38	1355.74	1289.56	4.9%	0.00	38.6%
24	245.82	236.72	3.7%	0.00	45.6%	39	788.06	783.53	0.6%	0.00	42.4%
25	878.85	872.49	0.7%	0.00	40.0%	40	384.11	379.20	1.3%	0.00	41.8%

value of LB_H and Δ are listed in the 8*th* and 9*th* columns of Table 5. Although our LBs are less than the Gurobi LB for small instances (1-10), our LB is always greater than the Gurobi LB for medium-size instances (11-40), resulting in an average Δ which is about 51.5%. As in Hoogeveen, Van Norden, and van de Velde (2006), we struggle to find a good LB, whether from Gurobi or LB_H. Nonetheless, using the LBs allows us to normalise the performance metric to a consistent measure relative to the value found. Therefore, we will use our LB for further assessment of heuristics.

5.2. *Comparing heuristics*

We compute the objective value and running time for 30 test runs of each instance. Let $TWCT_H$ denote the average objective value achieved by applying each heuristic listed in Table 1. The gap, $Gap_H = ((TWCT_H - LB_H)/TWCT_H)\%$, is computed to assess heuristics solution quality. Table 6 list the minimum, mean, and maximum objective value, the Gap_H, and the computational time of all 40 instances for each heuristic. For Instances 1-10, heuristics *C1R*, *C1W* , and *C1J* found optimal solutions while other methods found near-optimal solutions. For Instances 11-40 except Instance 15, the heuristics provided better objective values than Gurobi (1-hour time limit). All proposed heuristics found feasible solutions for Instance 32 within 24 seconds. The Mood's median test, a non-parametric method for testing whether two or more populations are from the same distribution based on their medians, is used. It provides 95% confidence intervals (CIs) for medians to compare the heuristics testing the following null hypothesis: all medians of all Gap_H are equal. The p-value of ~ 0.0001 (< 0.05) indicates that we can reject the null hypothesis. The CIs of Gap_H for all heuristics, shown in Figure 4, show that product-splitting method *C1* is better than the other two methods *C2* and *R*. For the 40 instances, sequencing methods *RK* and *WSPT* perform almost the same, but are better than *JR*. These 40 instances are small- and medium-size problems in terms of the maximum number of products considered (six). The permutation of products is less than the number of iteration (1000) used by each heuristic method. *RK* has a high probability of performing well by enumerating all products sequences.

To further compare the performance of *RK* and *WSPT*, we generate two larger problems having six orders with 15 products (O6) and 10 orders with 27 products (O10). The weight of orders is $\sim DU[1, 10]$. The number of machines is set as 6 in stage 1 and 18 in stage 2. Other parameters are as specified in Tables 2 and 4. For each instance, we run 30 replications using each heuristic. The 95% CIs for medians obtained from Mood's Median test for instances O6 and O10, shown in Figure 5(a) and (b), clearly show that *WSPT* out performs *RK* for these larger instances.

5.3. *Seeding Gurobi with heuristic solutions*

We seed *C1W* solutions in Gurobi to investigate if/how seeding improves MILP effectiveness. The run time limit is set to 1 hour. Table 7 shows the input (V_{in}) and output (V_{out}) objective values for Instance 11-40. It turns out that using these seeds could improve Gurobi's performance such that it provides better solutions for 27 instances including Instance 32—the average (maximum) improvement is 4.8% (27.5%). Some LBs returned after seeding are improved (see the 6*th* column in Tables 5 and the 5*th* and 11*th* columns in 7). Using our calculated LB to evaluate the performance of this hybrid method, the gaps ($Gap_{GH} = ((V_{out} - LB_H)/V_{out})\%$) are listed in the 6*th* and 12*th* columns in Table 7. Since the computational time of $C1W$ is less than 20 seconds, the total running time of this method can be considered as 1 hour which is acceptable for the coating company.

6. Conclusions and future research

We study an integrated batching and lot streaming problem with variable sublots, incompatible product families, and sequence-dependent setup in a 2-HFS. An MILP model with two valid inequalities are presented for this problem wherein the number of sublots for each product, the size of each sublot, and the production sequence for each sublot are determined simultaneously to minimise TWCT. Our study shows that both MILP and MILP_{2VI} can provide optimal solutions for small instances but cannot be solved to optimally for medium-size instances, within the operating time-frame used by the manufacturer. Also, feasible solutions cannot be obtained for one problem instance, which suggests the need for heuristic development.

We evaluate nine heuristics, each supplemented with RLS. Heuristics $C1R$, $C1W$, and $C1J$ found optimal solutions for Instances 1-10. All heuristics find better solutions than time-limited Gurobi solver for Instances 11-40 except Instance 15. Further, the computational times of all heuristics are much shorter than the time required by Gurobi. Our statistical analysis leads us to conclude that $C1W$ is the best heuristic method among all nine methods studied. Finally, a hybrid method which seeds Gurobi with $C1W$ solutions improves solutions by 4.8% within the allowable computation time—this method enables companies to make production scheduling decisions effectively.

There are several possible extensions to our research. First, the model could be extended to a multi-stage hybrid flow shop. Larger multi-stage problems are more typical in practice. Another direction for further research is to consider other machine environments such as job shop. Furthermore, multi-objective lot streaming problems integrated with batching can be investigated in future research.

Disclosure statement

No potential conflict of interest was reported by the authors.

ORCID

Shasha Wang http://orcid.org/0000-0002-1783-2677

References

Adonyi, R., G. Biros, T. Holczinger, and F. Friedler. 2008. "Effective Scheduling of a Large-Scale Paint Production System ." *Journal of Cleaner Production* 16 (2): 225–232.

Biskup, D., and M. Feldmann. 2006. "Lot Streaming with Variable Sublots: An Integer Programming Formulation." *Journal of the Operational Research Society* 57 (3): 296–303.

Bożek, A., and F. Werner. 2017. "Flexible Job Shop Scheduling with Lot Streaming and Sublot Size Optimisation." *International Journal of Production Research* 56 (19): 6391–6411.

Cheng, M., N. J. Mukherjee, and S. C. Sarin. 2013. "A Review of Lot Streaming." *International Journal of Production Research* 51 (23-24): 7023–7046.

Defersha, F. M., and M. Chen. 2010. "A Hybrid Genetic Algorithm for Flowshop Lot Streaming with Setups and Variable Sublots." *International Journal of Production Research* 48 (6): 1705–1726.

Defersha, F. M., and M. Chen. 2012. "Jobshop Lot Streaming with Routing Flexibility, Sequence-Dependent Setups, Machine Release Dates and Lag Time." *International Journal of Production Research* 50 (8): 2331–2352.

Erramilli, V., and S. J. Mason. 2006. "Multiple Orders Per Job Compatible Batch Scheduling." *IEEE Transactions on Electronics Packaging Manufacturing* 29 (4): 285–296.

Erramilli, V., and S. J. Mason. 2008. "Multiple Orders Per Job Batch Scheduling with Incompatible Jobs." *Annals of Operations Research* 159 (1): 245–260.

Feldmann, M., and D. Biskup. 2008. "Lot Streaming in a Multiple Product Permutation Flow Shop with Intermingling." *International Journal of Production Research* 46 (1): 197–216.

Han, Y., D. Gong, Y. Jin, and Q. K. Pan. 2016. "Evolutionary Multi-Objective Blocking Lot-Streaming Flow Shop Scheduling with Interval Processing Time." *Applied Soft Computing* 42: 229–245.

Hoogeveen, H., L. Van Norden, and S. van de Velde. 2006. "Lower Bounds for Minimizing Total Completion Time in a Two-Machine Flow Shop." *Journal of Scheduling* 9 (6): 559–568.

Kurz, M. E., and R. G. Askin. 2004. "Scheduling Flexible Flow Lines with Sequence-Dependent Setup Times." *European Journal of Operational Research* 159 (1): 66–82.

Li, D., X. Meng, Q. Liang, and J. Zhao. 2015. “A Heuristic-Search Genetic Algorithm for Multi-Stage Hybrid Flow Shop Scheduling with Single Processing Machines and Batch Processing Machines.” *Journal of Intelligent Manufacturing* 26 (5): 873–890.

Lin, R., and C. J. Liao. 2012. “A Case Study of Batch Scheduling for an Assembly Shop.” *International Journal of Production Economics* 139 (2): 473–483.

Logendran, R., P. deSzoeke, and F. Barnard. 2006. “Sequence-Dependent Group Scheduling Problems in Flexible Flow Shops.” *International Journal of Production Economics* 102 (1): 66–86.

Mönch, L., and S. Roob. 2017. “A Matheuristic Framework for Batch Machine Scheduling Problems with Incompatible Job Families and Regular Sum Objective.” *Applied Soft Computing* 68: 835–846.

Mukherjee, N. J., S. C. Sarin, and S. Singh. 2017. “Lot Streaming in the Presence of Learning in Sublot-Attached Setup Times and Processing Times.” *International Journal of Production Research* 55 (6): 1623–1639.

Olson, J. R., and M. J. Schniederjans. 2000. “A Heuristic Scheduling System for Ceramic Industrial Coatings.” *Interfaces* 30 (5): 16–22.

Pan, Q. K., M. F. Tasgetiren, P. N. Suganthan, and T. J. Chua. 2011. “A Discrete Artificial Bee Colony Algorithm for the Lot-Streaming Flow Shop Scheduling Problem.” *Information sciences* 181 (12): 2455–2468.

Potts, C. N., and M. Y. Kovalyov. 2000. “Scheduling with Batching: A Review.” *European journal of operational research* 120 (2): 228–249.

Pranzo, M. 2004. “Batch Scheduling in a Two-Machine Flow Shop with Limited Buffer and Sequence Independent Setup Times and Removal Times.” *European Journal of Operational Research* 153 (3): 581–592.

Rashidi, E., M. Jahandar, and M. Zandieh. 2010. “An Improved Hybrid Multi-Objective Parallel Genetic Algorithm for Hybrid Flow Shop Scheduling with Unrelated Parallel Machines.” *The International Journal of Advanced Manufacturing Technology* 49 (9): 1129–1139.

Shahvari, O., and R. Logendran. 2018. “A Comparison of Two Stage-Based Hybrid Algorithms for a Batch Scheduling Problem in Hybrid Flow Shop with Learning Effect.” *International Journal of Production Economics* 195: 227–248.

Shi, H., and F. You. 2016. “A Computational Framework and Solution Algorithms for Two-Stage Adaptive Robust Scheduling of Batch Manufacturing Processes under Uncertainty.” *AIChE Journal* 62 (3): 687–703.

Tang, L., H. Xuan, and J. Liu. 2006. “A New Lagrangian Relaxation Algorithm for Hybrid Flowshop Scheduling to Minimize Total Weighted Completion Time.” *Computers & Operations Research* 33 (11): 3344–3359.

Trietsch, D., and K. R. Baker. 1993. “Basic Techniques for Lot Streaming.” *Operations Research* 41 (6): 1065–1076.

Wang, S., H. Gangammanavar, S. D. Ekşioğlu, and S. J. Mason. 2019. “Stochastic Optimization for Energy Management in Power Systems with Multiple Microgrids.” *IEEE Transactions on Smart Grid* 10 (1): 1068–1079.

Yin, Y., T. C. E. Cheng, D. Wang, and C. C. Wu. 2016. “Improved Algorithms for Single-Machine Serial-Batch Scheduling with Rejection to Minimize Total Completion Time and Total Rejection Cost.” *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 46 (11): 1578–1588.

Zandieh, M., and E. Rashidi. 2009. “An Effective Hybrid Genetic Algorithm for Hybrid Flow Shops with Sequence Dependent Setup Times and Processor Blocking.” *Journal of Industrial Engineering* 4 (8): 51–58.

Zhang, B., Q. K. Pan, L. Gao, X. L. Zhang, H. Y. Sang, and J. Q. Li. 2017. “An Effective Modified Migrating Birds Optimization for Hybrid Flowshop Scheduling Problem with Lot Streaming.” *Applied Soft Computing* 52: 14–27.